TONE CLOCK THEORY EXPANDED:
CHROMATIC MAPS I & II

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- A NEW GUIDE TO THE CHROMATIC SYSTEM

Jenny McLeod

Studies of characteristic reading patterns show New Zealanders are uncommonly fond of books that categorise and classify information. To this extent at least, I am a true kiwi.

But notwithstanding, music shall reign. This little book is dedicated, with deepest respect and affection, to the memory of my teacher and friend Olivier Messiaen, who held that one must love the notes one writes.

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* now the New Zealand School of Music
** now Creative New Zealand

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Preface

The purpose of a composer in learning to know and understand the laws and properties of the chromatic system is necessarily somewhat different from that of a pure theorist or mathematician.

'The scientist has a certain *attitude* toward nature,' writes the American philosopher Arthur Young, in his fascinating contribution to the theory of process, *The Reflexive Universe* (1976). 'He is preoccupied with the study of law—and having discovered it, he holds it sacred. The inventor too must discover law, but this is not his goal. He has his mind set on something he wants to achieve, to fly, for example, or to communicate without wires. So he must both learn the law and then apply it, which involves a turnabout, a change of direction. The law is essentially restrictive, it limits the possible; but when it is stated objectively, we may find that it can be turned about and will, through its very certainty, provide the means by which our end can be achieved.'

In order to develop the mental powers he thought essential for his work, Young decided he first needed to invent something—but what? He went to the patent office, looked up all the failed projects, and decided a helicopter that worked would be good; over the next nineteen years, he designed and eventually built the prototype of the successful Bell helicopter —then he went back to his theory of process. It was for Young, as it is for the composer, a matter of 'learning to *use* a law instead of being blocked by it', of approaching it 'as the *agency* of free will rather than inherently in conflict with it.' Ideally, a law can *help* us to be creative, and really, for a composer, that is the only *point* of any law.

To a considerable extent, as composers, we today create our own laws, our 'own personal' musical systems. Hence it may be thought that we have only ourselves to blame if we find ourselves 'blocked' by them, for in this case it is surely our prerogative simply to change the laws or to break them—to burst free of them at any moment we choose. The latter, however, is easier said than done, and may even be a prospect fraught with peril for composers of artistic conscience. Most of us prefer to work from some logical departure point, to contain a piece within certain chosen limits. At the same time, it can be all too easy to be dominated by a 'system' (or simply by a fashion: the semblance of a system), even a system or a style of one's own creation—and even easier to be dominated by systematic or stylistic assumptions that are inherited or habitual and unconscious.

In reality, none of us creates completely 'from scratch' our own laws and systems, for this is not possible. We do not exist in a vacuum. Even the greatest of us have looked to other, earlier composers for some help, some ideas as to how viable music might be created. Only a very few, moreover, have ever developed any significantly new language or techniques (not necessarily a sign of supreme greatness in a composer, however, for others who have not done this quite so noticeably are

sometimes greater). Amongst them, in the twentieth century, I would number Schoenberg, Webern, Messiaen, Xenakis, Boulez, Stockhausen, Carter, Babbitt and Schat—and along more personal lines, Debussy, Stravinsky, Bartok.

The question arises: how new, really, are the so-called systems or methods of these composers (where they had any)? Were these systems discovered or were they invented?—the old question that is still asked and argued about mathematics. And to what extent does any system really make the actual *music* in any case—given that a composer worth their salt can make music from a couple of tin cans, let alone from an entire chromatic realm, of whose actual nature he or she may well be (and may indeed prefer to remain) relatively ignorant. No system comes with any guarantee of quality; there are no infallible recipes—we all know that. The spark of life we seek is not so readily summoned, and may even be choked off by too much system. In such matters it is well to be wary, to feel one's way cautiously.

While these questions may ultimately be unanswerable, they are worth considering. On one hand, we find Schat asserting that the *Eroica* was 'written as much by E-flat major as by Beethoven', and on the other hand, Messiaen: 'I consider that the terms "tonal", "modal", "serial", and other words of this kind are illusory, and that their use constitutes a lie: they are phenomena that probably never existed. They were exploited in books because one could draw up beautiful theories with lovely chart-summaries, but those are things of no importance, of which composers have finally taken little account. If the only quality of Mozart's *G minor Symphony* consisted of its being in G minor, do you think there would be anything left?...It is written in G minor, but what is important is its thematic and harmonic material, its rhythms and accentuation.'

Clearly Messiaen is right, in that the music itself is what really counts[1]—and in the matter of charts, he might have wished to renounce his old pupil utterly, for I must be the most inveterate and unashamed maker of charts that ever lived. But I do not mistake their importance: essentially I made them for myself, to be absorbed into my bones and then forgotten (absorbed in general, that is, not in absolutely all of their glorious detail, which is simply there to be looked up if and when one needs it, much as one might consult a book of maps whilst wandering through a world). The least scrap of real music (i.e., music with a spark of life) is more important than anything in this book. With that, where anyone else's music is concerned, all theory may fly out the window for all I care.

Myself, I've always preferred to compose more intuitively than systematically—but I like to compose intuitively from a well-prepared base.

[1] Though perhaps we should here also recall Stockhausen, who asserted that the 'model character' of a work is for him more important than any realisation of it in sound—an approach from which some may well feel his music has sometimes suffered, but which is perfectly understandable in view of the numerous strikingly original 'model characters' he has actually created.

Experience with many different kinds of approach has convinced me that freedom is illusory in the absence of a previous thoroughly-absorbed discipline which has become so instinctive, so much 'second nature', that it comes spontaneously. Without the increased awareness that accompanies patient and willing self-discipline, I find one is a slave, unconsciously, of one's unconscious mind. That may be fine (i.e., safe) for some, but not for me. I approach my unconscious more as a partner in the creative enterprise. Left completely to itself, it has a habit of landing me eventually in untold trouble, but channelled and directed—given specific but sufficiently broad directions and limitations—it has (on a good day) much better and fresher creative ideas than any I can manufacture consciously. With me, as I imagine with many of us, it's a pretty constant two-way affair. At each step, I do consciously as much as I can (as much as I feel is possible or necessary at that point, in the way of 'pre-programming' or conscious attempts), then I more-or-less step aside and let my unconscious dwell on and absorb what I have given it, waiting for it to come up with some musical ideas. Material that is arrived at intuitively I then assess, reject or accept, consciously analyse, perhaps elaborate upon and develop. This alternation between the two of us, between my conscious and unconscious minds, may take place from moment to moment (in the heat of actual composing), or from minute to minute, hour to hour, or any other time-span ('wave-length', almost)—sometimes years, where large-scale planning and contemplations are concerned.

I pursued a knowledge and understanding of the chromatic system partly in order to improve the *basis* of my intuition, to increase my technical range, so as to operate less from old habits that were to some extent at least acquired unconsciously, and more from what is really possible—in short, to enlarge, refresh and improve my 'second nature', to develop some better habits, and to get a real feel for the chromatic territory. I like the process of composition to be a journey of discovery (for me, as well as for the listener later on), but a journey where I always have a map in my pocket or in my mind, so that at any point I can always more or less know, or on reflection be able to identify, 'whereabouts I am' in the larger system—and thus can also realise (or remember, or look up) what else lies in the neighbourhood, and so have some idea of where I might conceivably move on to from there.

The matter of finding and defining some appropriately broad first limitations within which one's unconscious can operate successfully and spontaneously has been for me a major concern. The kinds of limitation of which I am speaking are the same, in their essential *nature*, as those which for Mozart or Beethoven were provided by 'G-minor' or 'E-flat major' (and also, for Messiaen, by his modes of limited transposition, for that matter), so there is a sense in which Schat too is right—though undoubtedly Beethoven and Mozart were just as powerfully motivated by the desire to *avoid* doing everything that more commonplace composers did with G minor and E-flat major, to avoid the cliché generally inherent in too

explicit, too naive expressions of the paradigm (the deep structure) itself. But for all that, G minor or E-flat major are not exactly 'nothing'—and nor are the modes of limited transposition (in fact they constitute one of the most significant and central areas of the chromatic system). I doubt that Messiaen himself really believed this either, or he would scarcely have bothered to write his *Technique de mon langage musicale*. As functional tonality or as chromatic mode, they represent broad first limitations within which the composer's unconscious has demonstrably been able to function very successfully. They may seem to be little in themselves, indeed next to nothing—but the unconscious does not necessarily *need* so very much to 'get it going'. Because of the exceptional mathematical properties involved, both the old tonality and Messiaen's modes are also extremely *potent* limitations, to which, on the personal level, the composer's unconscious can respond (in the case of the former, has already responded, to the point of exhaustion) in an enormous variety of different ways depending on the individual composer. That Messiaen happens to have had a completely unique personal style has in this respect misled many of us about the nature of his modes, for they are not just some sort of 'exotic scales' of his own that he dreamed up purely by chance, though he did indeed come to them gradually and intuitively at first. In fact, like the old tonality and like everything else this book focuses upon, they are essentially supra-personal, and have a vast potential for use by others in ways completely different from his, as some have already recognised.

In a very real sense, western music as we know it today began with the advent of equal temperament. This transformed the chromatic scale and the full cycle of fifths, with all the concomitant possibilities of key modulation, from theoretical abstractions into musical realities. With its subdivision of the octave into a given number of uniform parts, equal temperament introduced the significant property of homogeneity within the realm of the octave—the same homogeneity, in fact, that the octaves themselves had already introduced, at the larger level, into the realm of musical pitch as a whole (the simplest sort of musical fractals, if you like). This homogeneity dramatically increased the range of an already-existing musical operation, transposition, making it possible to transpose any group of notes from any point in the system to any other point.

The principle of equal temperament also brought with it a new potential for democracy—a potential not fully realised until Schoenberg—since it made every note theoretically equivalent to every other note[2]. It *also* brought with it all the many wondrous properties of the so-called 'dodecahedral group D2' (the name, in group theory, of the particular mathematical group formed by the now closed twelve-note chromatic system). Whilst I know little of mathematics, I do well know that, in musical terms, these various mathematical properties exert, for me at any rate, a great fascination and attraction, as much for my ear as for my mind.

[2] In practice, of course, no two notes are ever completely equivalent.

The structure and sub-structures of the chromatic system itself, the different 'behaviour' of its various groups, both in their inherent properties and in the way they 'people' our music—all of this constitutes a real 'sociology of the tones', as I have called it here, which has become for me a source of endless interest, as a society of 'chromatic individuals' whom one can come to know increasingly as old friends.

Approaching the dawn of the twenty-first century, we are no longer in the same fearful chromatic dark that Schoenberg found himself plunged into—or if we are, we do not need to be. We are now at a point where composers, and particularly younger composers, are coming to be much more interested in, and to understand rather better, the real extent and nature of the chromatic system, the possibilities it suggests, the things it will 'do best'.

Equal temperament has excluded or blocked (for equal-tempered instruments) the possibility of pure 'natural' interval ratios, whilst simultaneously opening up a whole new range of possibilities. It can become an *agency* of the composer's free will (as it did for J. S. Bach, for example), making possible—indeed of its very nature *suggesting*—the exploration of certain things, the creation and achievement of new goals that formerly were not possible. But it can also come into conflict with, or block, the composer's free will (depending on the desired goal), since it also makes impossible certain other things that formerly were possible. Which of these it will do is largely dependent on the composer's attitude: 'What can I *do* with all this?' as compared with 'What does all this *prevent* me from doing?'

In developing the present ideas, I have kept both of these questions steadily in mind. Where I see no possible compositional use for something, I do not bother with it. If I like someone else's theory, but feel it is applied in too limited a way, I simply extend it—thus you will find my version of the tone-clock theory, for example, a good deal more expanded than Schat's, who invented it. When theory prevents me from doing something that I might want to do, or that I feel I should at least be *able* to do (even though I may never want to), I do not change myself to adapt to the theory—I change the theory, to allow for my potential needs.

In this way, as I proceeded, my thinking became more and more inclusive and less and less of 'a theory', as something 'different from' or 'opposed to' other theories. And in fact, it became more and more a study of the chromatic system itself, whose inherent nature and properties, as I came to see ever more clearly, gave rise in the first place to so many of the theories and techniques of others that most interested me, and also to the further ideas and techniques that came to light, intuitively at first, in my own music.

I sensed at the outset of my explorations, and shall begin to demonstrate here, that these various techniques form the 'bones' of a larger whole—that in fact none of them is 'opposed' to any other and it is not a matter of having to choose one over another or to join any particular

'camp'; that all the various areas they cover are linked to one another by having some important feature or principle in common (so that it is possible to move perfectly consistently from one area to another or to combine any of the different areas); and that *together* they form a whole great network which is not 'a' system so much as *the* system: the chromatic system. It is this that is our 'common language'—the most obvious of truisms, perhaps, but a statement that becomes much more meaningful in the light of the fact that the chromatic system turns out, surprisingly, to be quite a lot *smaller* than any of us probably imagined when we first entered it. So that, given time, effort and practice, it is quite possible for us to absorb its basics more-or-less in their entirety—and this is the necessary condition for a common language.

Thus if you use the twelve equal-tempered notes (and if you care to look), you will find yourself in here somewhere, though the angle of approach may be unexpected, and though there are no doubt still new chromatic techniques to be discovered and incorporated as well. If I am right, however, these will be more additional than essential. As I see it, we have already arrived at the point where the various main techniques taken together form a larger whole without any serious gaps[3].

Included within the chromatic system are the old tonality, all poly-tonality and atonality, Schoenberg's (Webern's, Berg's—also Hauer's) dodecaphony, Messiaen's modality, Babbit's combinatoriality, Boulez's frequency multiplication, American set theory, Xenakis's sieve theory (in its twelve-note form) and Schat's tone-clock theory, all these representing what I see as the most important theoretical developments in respect of the chromatic realm, and representing different aspects, each essential to the whole, of a larger integrated and exceedingly rich melodic-harmonic system.

To my mind, therefore, we can forget about hard-edged musical divisions (although what is worn out from over-use is still worn out, and questions of choice and taste remain as important as they have always been). In all these theories and methods, it is the chromatic system first—thus ultimately equal temperament—that has made them possible, or that has brought them forth or suggested them as possibilities. Without equal temperament, none of them could have existed; and furthermore, most of them could also still exist in principle, though with a different form and sound, in any *other* system of equal temperament. To this extent at least (and inasmuch as our present chromatic scale sounds the way it does), the system does 'determine' the music, as Babbitt was probably the first to observe in respect of the structure of twelve-note rows—but it does not *create* the music, thank god. Only the composer can do that. We are after all only talking about theory, about simple deep structures, and not about music.

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[3] I filled in myself the few I saw, though not all these 'fillings' are to be found in the present volume.

This book thus attempts a new interpretation and synthesis of what is already known, together with a few developments or extensions that have suggested themselves. There is also a good deal of detailed chromatic information in the aforesaid charts which—for all that I attach no great importance in itself to such dogged accumulations of fact[4]—has not hitherto been available, or has in some cases been presented earlier from a somewhat different point of view.

Very likely, to offer up such material is also to hasten the eventual demise of the chromatic system itself. By the time all the significant chromatic facts have become common knowledge, and are taught automatically in our musical institutions, the chromatic system may well have become as academic and as exhausted for serious purposes as the old diatonic tonality that is taught today. This prospect does not disturb me unduly, however. By then our indefatigable pioneers will probably already have invented or discovered some viable new musical system or systems (whose seeds may indeed already be with us). But even if not, the human race, so far as we know, has never been without music. The presence or absence of a conscious musical 'system' has not changed this—though it may have shaped music's outward features—and I doubt that it ever will.

Meanwhile, the chromatic era is still in its youth. Potentially there lies ahead of us yet a 'golden age' of chromatic maturity, an age in which composers in general may, through patient training and practice, become much more experienced in consciously differentiating and identifying the basic pitch-combinations and our responses to them, and thence, perhaps, more accomplished in manipulating the rich and radiant world of colours, the *chromos* from which our chromatic system takes its name. (Indeed, we shall *need* such an age, if the larger body of composers is ever to 'catch up' with the few great chromatic exponents of the present age.[5])

As for music herself, I expect she will go her way serenely untroubled by any of this, and continue to smile upon whomever she chooses.

Jenny McLeod

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[4] Though actually I enjoyed covering all this ground (and a good deal more besides) *without* the aid of a computer, and finding out for myself exactly what the story is—to the amazement, and sometimes the consternation, of those looking on.

[5] Messiaen once remarked (rather uncharacteristically) that it would be two hundred years before his harmony was properly understood; and even today, nearly forty years after *Le marteau sans maître*, the harmonic art of Boulez is still understood by relatively few.

Part One Chromatic Map I: Intervallic Prime Forms

On Terminologies

In communications about music just as in music itself, it goes without saying, there must be room for flexibility, room for the personal, even the idiosyncratic, room at any rate for some exercise of the creative imagination. On the other hand, it does help if we can all more-or-less understand one another when we are speaking technically about music. To this end musicians have generally finished up agreeing about the meaning of the technical terms they have found most indispensable. There is no disagreement, for example, about the meaning of such terms as 'interval', 'mirror inversion', 'symmetry', 'transposition', and so on: we all know what they mean, they do not have to be redefined every time we use them.

Like music itself (and like language), musical terminology has a living quality, it is not something static. As music itself has evolved with time, so its technical terms have necessarily also evolved. At the 'frontiers', new musical concepts are born, along with their new technical terms. Nearer the centre, whatever has proved redundant, impractical, or plain cranky, has faded into obscurity, and yesterday's new concepts, now older and well-established, become more refined and differentiated. Still older concepts become exhausted, are superseded and in their turn die out–which is not to say that they might not perhaps be reborn, transformed, subsumed in some newer idea later on. Nor can the pace of this process of change be forced, for musicians must feel at home with the terms they use (although no doubt they will feel more at home with the notes themselves). They must be able to live and work with them; in short, I hold that they must *like* them. Not only musical notes, but also the ways in which we talk about them, have the power to attract or repel us.

The difference between an appropriate and an inappropriate terminology (it may well be, for one and the same concept) is not negligible. Although the best and most long-lived technical ideas have on the one hand evolved *from* music itself, they have on the other hand also profoundly influenced its subsequent course. Technical ideas and terminology are the only means by which composers can think *about* music (as opposed to 'thinking music', or improvising it, or simply feeling it)–and the ways in which we think about, as well as feel, music are inevitably crucial to the kinds of music we will compose.

Hence terminologies can be significant. We cannot think clearly about things for which we have no names, for instance. This is an extremely important point. We could not think about music as we do at present, for instance, had not various predecessors at some stage given names all to the notes, the intervals, the chords, the scales, the keys, and so on. To a considerable extent, the thinking of composers is conditioned by

the available terminology (just as *all* of our thinking is conditioned at the deepest level by the structures and concepts of the language in which we think)—and this continues to be true even when composers, groping for new ways to think about music, have invented new terminologies for themselves.

Moreover, a potentially fertile technical idea that is new can communicate the wrong message, can even perish, because it was expressed insufficently clearly or congenially *in words*. Boulez's 'frequency multiplication' (and most of the rest of his serial theory) suffered this fate, for example—only to be resurrected in a new guise, however, in his pupil Schat's concept of 'steering'. Thus there are certainly terminologies that can get in the way of the concepts they represent. As the most extreme instance of this in my own experience, the language of set theory so instantly repelled me, I regret to say, that it was almost twenty years before I could bring myself to investigate it at all. There are other terminologies that impress one as apt and appropriate for the concepts they represent, terminologies that stimulate rather than frustrate the creative imagination, that really strike one as musically orientated. (I find this with Messiaen and Schat, for instance, whose technical terms seem to me perfectly suited to the concepts they represent.)

Such responses are subjective, granted, but they are important—and set theory, indeed, is the case in point, for it turned out that I was far from alone in my first instinctive response to this. Thus it came about that, by its terminology alone, the most complete account until now of the chromatic system automatically barred itself from any possibility that it would ever be understood by 'ordinary mortals', namely, by the large majority of musicians with no particular mathematical inclination or training. Indeed, set theory was so unreadable to most of us that we did not even realise this was what it *was* (for apart from this, it is hardly a theory at all—it is much more a descriptive codification[1]). In the process, it has more-or-less set apart and 'claimed for itself' (though I do not say deliberately, for this was surely not intentional) something that rightfully belongs to all of us.

The Chromatic System

What is the chromatic system, then? Do we actually know very much about it? How familiar are we with its fundamental groups and their properties? Commonsense suggests that in order to make the best possible use of a system, it could be helpful to understand its properties.

The charting of the full extent of the chromatic universe, signified by the independent discovery of all the possible chromatic groups, earlier this

[1] The 'Kh complex' theory (Forte:1973) concerns only one kind of relationship (group A is a subset of group B, whilst B's chromatic complement is a subset of A's complement). This is interesting enough as far as it goes but cannot stand on its own, since groups of notes may exist in many different kinds of relationship with one another.

century, by Alois Hàba, Nicholas Slonimsky, Joseph Schillinger and Elliott Carter[2] respectively, represented something of a milestone, one might think, in our chromatic knowledge. Yet even today this is still not widely known. One reason is no doubt because these pioneers recorded their computations, as we might expect, in traditional stave notation, and in this form, strangely enough, the chromatic groups are actually far from easy to assimilate.

Nevertheless, the remarkable fact that the thousands of possible chromatic combinations reduce to only 222 basic groups (not counting the 12-note mother-group) will surely be significant for chromatic education in the future. Such a comparatively small number of forms is bound eventually to be learned and known back to front by musicians as a matter of course.

The chromatic groups subsequently received their most widely-known systematic classification in Allen Forte's classic set-theory text *The Structure of Atonal Music*(1973), where they are called 'pitch-class sets' and are each allotted a specific 'prime form'[3]. Their association there with integer notation and formulas has meant, however, that those for whom the mathematical approach has small appeal are still largely unaware even that all of the possible chromatic groups are known, to say nothing of what these groups actually are.

Needless to say, it is harder still for us to think about things when we do not even know they are there, let alone have names for them.

The Intervallic Prime Forms

In an attempt to remedy this situation, I offer here a new systematisation of the chromatic groups, namely, the intervallic prime forms (IPFs), which involve no integer notation or formulas and require no knowledge of set theory as such. A few of us in this part of the world have been working for several years with the present chart, which seems to serve the purpose well enough, though there is always room for improvement. All of the chromatic groups are included, apart from the 12-note group, whose prime form is the chromatic scale itself (i.e., the 'universal set', in set theory) and its complementary partner (the 'empty' or 'null' set), and the single 1-note group with its 11-note partner (both transposable twelve times). I have adopted Forte's order for the chromatic groups (as they appear in his Appendix 1, pp179-181) and have also included the intervals and the decads[4], categories he did not include there.

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[2] See Perle (1981) and Schiff (1983).

[3] See Appendix III, p110, if you are unfamiliar with the concept of prime forms.

[4] Since the term 'dyad' is often used to designate an interval, or group of 2 notes, I see no reason why the term 'triad' should not be extended to include any chromatic group of 3 notes, similarly 'tetrad' for a group of 4 notes, and so on ('pentad', 'hexad', 'heptad', 'octad', nonad' and 'decad'), in preference to the ungainly and somewhat illogical 'trichord', 'tetrachord', 'hexachord' ('monochord'?) etc., particularly when the last three also have other older technical meanings.

The IPFs are not simply a literal intervallic translation of the pitch-class set prime forms, however; in fact they were not developed from the PC sets at all. I arrived at my own understanding of the chromatic system via Schat's tone-clock theory[5] and Messiaen's modes, and my own extrapolations from these. That is, I came to it above all by way of the musical sounds, the actual harmonies, that most attracted and fascinated me, seeking a greater understanding of why they should produce such powerful effects, and also seeking possible ways in which the related pitch-techniques of Messiaen, Boulez, Xenakis and Schat might be combined and extended compositionally. In the process of expanding the tone-clock theory, I took from the outset the same intervallic approach that Schat did with the chromatic triads, and simply compressed and extended this to designate the larger chromatic groups as I worked them out. This was some years before I turned my attention seriously to set theory, so it happened that I arrived at the same ground as the latter quite independently, and by a completely different route.

When I did eventually go to the not inconsiderable pains of figuring out exactly what set theory was, I recognised there a different, sometimes bizarrely 'alien', version of what I already knew very well for myself—realising at the same time that I had seen things that the set theorists had not, or in ways that they had not, precisely because I am a musician and not a mathematician-musician. In fact, I was dumbfounded at 'what they had done' to my beautiful chromatic world—or rather, at the way they had *spoken* of it—for I could hardly conceive of any presentation *less* likely to communicate easily and amiably to most musicians. This did not prevent me, however, from immediately helping myself to anything I saw as practical, beneficial or useful in the set-theory approach, as you will see—and in fact this turned out to be more than I expected, so my antipathy has faded, to be replaced, even, by gratitude (if my colleagues will forgive such an ungracious acknowledgement).

I am not attempting here to 'replace' Forte's (or anyone else's) prime forms with my own versions, but simply to offer a practical alternative, or at least some pause for thought. His versions are, mathematically anyway, consistently organised and are an already-established point of reference where there is any argument. However, I do consider them in many respects unsatisfactory to work with practically.

Problems with Set Theory Prime Forms

One problem is that the endless, unrelieved integer notations make the groups too hard to distinguish from one another and thus on the whole impossible to remember, even though certain well-known groups do quickly become familiar (for instance, the diminished seventh as 0,3,6,9 or 1,4,7,10 or 2,5,8,11). Another disadvantage is that as soon as a chromatic group is transposed its integer notation changes. The so-called 'fixed-*doh*'

[5] See Schat (1993).

and 'moveable-*doh*' notations are a more recent innovation addressing this —although one may well smile at the rather unexpected reappearance of a solfège or tonic solfa term, in the ostensibly atonal context of integer notation (why '*doh*', why not just 'fixed or moveable *zero*'?). And so, having gone to the trouble of learning that C is '0', A-flat is '8', F-sharp is '6', and so on, one must then readjust to the fact that this is not necessarily so at all, and that '0', '8', etc., can in fact be any notes you like.

A further disadvantage of integer notation is that two of the numbers involved, 10 and 11, contain two digits each, thus necessitating some graphic means, such as a comma or a gap, to separate the individual numbers from each other, in order to avoid confusion. This takes up too much space, particularly when one is using them in score analysis. One or two set theorists have 'solved' this problem by the expedient of substituting X and Y for 10 and 11—or even A and B, in which case we have the anomaly that note 10, the equivalent (in fixed-zero notation) of B-flat, has unexpectedly acquired the new name A, and we are somewhat peculiarly right back where we started, with letters of the alphabet.

Despite these disadvantages of integer notation, however, there are still plenty of occasions where it is the simplest alternative (in working out a so-called 'twelve-by-twelve'[6], for instance, or permutating the notes of a group, or calculating subsets or invariant notes [notes in common] under transposition, and so on), so I am certainly not suggesting that we could do without it. Those of practical mind will simply use whatever works best in the circumstances.

On the other hand, if one uses Forte's set-numbers on their own, as he often does (i.e., the tags: 6.20, 7.35, 8.28, etc.), instead of naming all the actual integers of the prime form, the various groups are easily referred to, it is true, but one has lost touch with their actual structure. Thus wherever this structure happens to be less than memorable (which is more often than not) one has to keep on looking it up in his appendix to see what the group really is, which is tedious and time-consuming. One advantage of the intervallic prime forms is that the structure of a group is expressed in its name, so that it is not necessary to consult any further chart in order to find out what the notes of that particular prime form are. Nor, in this case, is it really necessary to provide any 'tag' at all, and in practice I mostly do not. However, the PC set-number can still be useful for identifying the chromatic complement, as well as for referring to a group in circumstances where its actual structure is for the moment of less importance, and not least for finding one's place quickly in the chart. These are all good reasons to keep the tags, which in addition provide an easy cross-reference to the Forte appendix.

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[6] A '12 x 12' is a matrix whose 12 rows and 12 columns show all the 48 forms of a 12-note series. The original form in all its transpositions is read from left to right, the retrograde from right to left, the inversion from top to bottom, and the retrograde inversion from bottom to top.

The other main problem with Forte's prime forms is that apart from the inclusion of their interval content (in an array originally from Martino[7], now standard set theory) too little account is taken, in my view, of the *structural* properties of the groups, which are really what distinguish one group from another in the mind of a musician, particularly a composer. Where a mathematician may understandably see no particular reason to prefer a symmetrical over an asymmetrical permutation, for example, and may thus be quite happy to take the most compact version, regardless of its structural features, as the 'best normal order' or prime form, the difference between a symmetrical and an asymmetrical form is for the composer, on the contrary, immediately one of the most fundamental structural properties that will attract his or her attention.

In formulating the intervallic prime forms, I found myself automatically applying a more extensive, and more specifically musical, range of criteria than the set theorists. This grew quite gradually and instinctively—in fact I did not stop to work out exactly what I had been doing until I later discovered how different their approach was from mine. Naturally, the IPF method too has its problem areas, but in coming to grips with these one is necessarily also involved in learning more of the particular nature of particular groups, which I find is a benefit rather than a disadvantage. In fact, it is a necessity, if we are really to become familiar with the chromatic system.

HOW TO READ THE IPF CHART

Those of us who in the past took a quick look at set theory and put it aside can be forgiven for thinking that it had little to do with music as we knew it, and thus no particular relevance for us—in short, that it was 'just another theory', and a mathematical one at that. Its comprehensive nature is not immediately obvious, nor the fact that *all* music in our equal-tempered system consists always and only of 'pitch-class sets'[8]. Moreover, Forte deliberately restricted his considerations to so-called 'atonal' music only, and made a determined attempt to exclude virtually all familiar musical terminology. Thus to anyone casting an eye over his pages of text, set theory does indeed appear to exist in a decidedly remote world of its own.

Alas, the present IPF chart, with its possibly dismaying array of numbers, may well give the same impression, I realise. Therefore I hasten to reassure the reader that these numbers simply represent a considerable amount of musical *information* in a highly compressed form, and must stress again that this is *not* a 'mathematical theory'. It is merely a reference source—a charting, a chromatic 'map'. All this has developed, it is true, along with my own musical theories, but I shall not be discussing

[7] See Martino (1961).

[8] So that 'those who look for sets' will of course always 'find them'. (This charge is more properly directed at those who look for *particular* sets and always find them.)

those here (although the various lines of thought that occupy me can already be gathered to some extent from the nature of my categories and definitions). At this point I propose simply to set out some terminology and fundamental concepts.

In order that this 'map' may be easily read (which is indeed within the reach of any musician), I shall now go through each of the columns in turn and explain exactly what the various numbers and symbols represent. (Certain of the footnotes contain additional explanatory information; for a full understanding, I suggest you do not read them later or pass them over, but rather that you read them in conjunction with the text.)

Existing Name Column

The first three columns of the present table are concerned with the different names for the chromatic groups that have emerged in the recent and more distant past—that is, existing names from the old tonality, from Messiaen's theory, labels from set theory and names developed after Schat's tone-clock theory. (These names are not all-inclusive, obviously, due to lack of space.)

One of the first things the newcomer to all this needs to realise is that the chromatic groups do not exist in some isolated realm of their own, divorced from all that we already know about music. All of the familiar note-groups are also found here, for these are all of the *possible* groups that can be formed from the twelve notes of the chromatic scale. This naturally includes the diatonic scale, also the wholetone scale, the pentatonic scale, the 'natural' triad, the augmented and diminished triads, as well as all of the larger chord-formations, and so on. So already there are some 'landmarks'; it is not all totally strange. In fact, a good way to begin with the chart is to try it out first with a few of these well-known groups (you may well be surprised to discover how much you did not know about the properties of the diatonic scale, even.)

'(Section of:)' refers to the fact that certain groups (in their most compact intervallic succession) are stepwise sections of familiar larger groups, such as the chromatic scale, the diatonic scale, etc. (shown likewise in parentheses in the first column)—thus they are the most obvious subsets of these groups. (Naturally it was not practicable or desirable to identify *all* of their subsets, since we are here including only the most salient features of the chromatic groups, those that will best help us to remember them.)

PC Set Column

'PC Set' signifies 'pitch-class set'. In this second column are the set-theory tags for the chromatic groups, as ordered and numbered by Forte. His actual integer notation for each set is not shown, since this can easily enough be checked in his book, and in any case we shall not need it.

Forte's numbers in parentheses alongside the set-number show the number of 'distinct forms' for that particular group, that is to say, the number of possible transpositions of that group. For asymmetrical groups which have a real mirror inversion, there are 24 distinct forms: 12 transpositions for the original and another 12 for the inversion. A symmetrical group can have only 12 transpositions at most, since it has no real mirror inversion (the mirror inversion is identical to the original). As we know, certain chromatic groups (for example, Messiaen's modes) have *fewer* than twelve transpositions. With one exception, the number 12 (or 2, 3, 4 or 6) appearing in parentheses alongside the set number *also* indicates that the group concerned is symmetrical (although with Forte's prime forms there is very often no indication of exactly *how* it is symmetrical)[9].

The meaning of the 'Z' followed by a number, included in square brackets after certain PC set numbers, is explained later, under the heading 'Interval Content Column'.

TC Name Column

The third column, which later in the chart becomes amalgamated with the IPF column alongside it, shows the tone-clock name(s) for the group (if it has any), after Schat's theory and my own subsequent development of this.

Twelve Chromatic Hours

The tone-clock names are based on Schat's conception that each of the twelve chromatic triads represents a different 'chromatic tonality' or colour—or 'hour', a term I use often[10]—dependent on its basic interval structure (see Appendix V, p114, for a stave-notation summary).

The twelve interval-patterns of the (prime-form) chromatic triads are referred to by the roman numerals I through XII (as it happens, Schat's numbering and order of the triads are exactly the same as Forte's[11]).

Triad I (1 +1 semitones [stacked]) is the triad of the 'first hour', triad II (1 + 2 semitones) is the triad of the 'second hour', and so on (see the IPF chart, p39, also p114, for the remaining triads).

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[9] The exception is the group 6.30, which, although it has only twelve transpositions, is *not* symmetrical. (This group, and *only* this group, has six transpositions in its original form, and another six in its inverted form, for reasons which will become clear if you study its intervallic structure, shown in the chart.)

[10] Schat has drawn the twelve triads systematically on a sort of clocks-within-a-clock-face (see Appendix IV, p113). This diagram, although beautiful, is not strictly necessary for an understanding of the affair, but the idea of musical 'hours' is in itself important and helpful. It provides a series of differentiated names for (and hence a new clarity to) a more-or-less subliminal awareness of intervallic textures that many of us have long had and worked with; moreover, the term 'hour' itself is appealing, appropriate, and capable of flexible use.

[11] Schat had no knowledge of or interest in set theory, however.

Hour-Groups
All groups with tone-clock (i.e., roman numeral) names belong in the general category of what I call 'hour-groups'.

An hour-group is any intervallic form (intervals stacked, without note-repetitions) which can be interpreted as belonging within a *single* chromatic hour. Not all hour-groups have specific roman numeral names, but only the simplest.

There are only ever two different interval classes, at most, in any hour-group. However, not every prime form containing only two different interval classes is necessarily in itself an hour-group (although if larger than a triad it will always consist of a *combination* of smaller hour-groups). Specific conditions apply to the interval combinations that constitute an hour-group (see Appendix II, p102, for a detailed discussion. This appendix shows all of the available hour-groups for each of the twelve hours. The same information is also contained in the IPF chart, but in terms of the hours this is more easily seen in the Appendix II table, pp106-109. See also pp114-5 for a stave-notation summary of what follows.)

Minor & Major Forms; Fundamental Configurations
Where a chromatic triad or larger hour-group contains two different intervals (as opposed to one interval repeated), I distinguish it as having either a 'minor' (m) or a 'major' (M) form.

A 'minor' chromatic triad, for example, has the smaller of its two intervals first in the prime form, and a 'major' triad has the larger interval first. Thus triad IIm^3, for example (or IIm, for short), is 1+2 semitones, whereas triad IIM is its inversion, 2+1 semitones, both found under the same heading in the IPF chart.[12] Hence we can speak of 'minor and major' triads of the second hour, the third hour, also the fourth, fifth, seventh, eighth, ninth and eleventh hours (the last being also the old 'natural' minor and major triads). This is a logical extension from, as well as a subsuming of, the old meanings of minor and major.

In hour-groups larger than triads, the minor/major concept becomes further extended: the intervallic structure of a major group has the same *fundamental pattern of changes*—i.e., the same configuration of (interval) alternations and repetitions—as its partner minor group, but the smaller and larger intervals *exchange places* (also true of the triads above)—and vice versa, naturally. If the fundamental configuration is symmetrical, then neither the minor nor the major form has a real mirror inversion, but if it is asymmetrical then both have an inversion.

For example, the second-hour minor symmetrical tetrad, IIm^4 (or ST IIm), is 1+2+1 semitones, while its major partner the symmetrical tetrad

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12 Strictly speaking, it is the retrograde inversion, since all our prime forms take a rising, rather than a falling form, but we shall call it the inversion, with the 'retrograde' being understood. In any case, these are merely the prime forms: in practice, the notes themselves may occur in any order, for a chromatic group is essentially an *unordered* group, a so-called 'collection'.

IIM^4 is 2+1+2 semitones—which is *not* the inversion, as was the case with the triads a moment ago, but is a different group found under a different heading. (The superscript number indicates the number of notes—*not* the number of intervals—in the group; for the sake of economy this is omitted in the case of the triads, so that a roman numeral on its own signifies a triad.)

Here the fundamental pattern or configuration of a symmetrical tetrad is clear—a first interval is followed by a second interval, and then by the first interval again. The roman numeral tells us the 'hour', that is to say, it tells us what the two intervals actually *are*. Which is the minor and which the major form is determined, as with the triads, by referring to the *first two intervals* of any such larger hour-groups, i.e., to the constitution of the first triad in the intervallic form: the minor form has the smaller interval first, and the major form has the larger interval first. Knowing this, and knowing the general structure of a symmetrical tetrad or a symmetrical pentad, say, the rest of the group can then easily be reconstructed.

Oedipus Groups & Subscales

Groups larger than triads that have specific roman-numeral names are the simplest larger chromatic groups, in which a single interval is repeated, or else two different intervals alternate, in the prime form. I call the latter the 'oedipus' groups (because they 'limp along'). For example (with arabics indicating intervals in semitones):

third-hour oedipus pentad:

minor form: c c# e f g#
1 3 1 3

major form: c e♭ e g g#
3 1 3 1 (= inversion of the minor form)

The minor and major forms of the *largest* oedipus group in the second, third, fifth and eighth hours respectively are always a cyclic permutation of the same group of notes, and are also always (oedipus) 'subscales', i.e., chromatic groups whose interval pattern if continued simply repeats the same notes throughout the octaves. E.g.:

fifth-hour oedipus subscale:

1 5 1 5 1 5 1
minor form: c c# f# g c' c#' f#' g'(etc.)
(major form): cyclic permutation (etc.)

5 1 5 1 5 1 5
major form: c f f# b c' f' f#' b' (etc.)

The largest group in the first, sixth, ninth, tenth and twelfth hours respectively is likewise always a symmetrical (homogeneous) scale or

subscale—namely, the chromatic scale, the wholetone (sub)scale, the scale of fourths, the (sub)scale of minor 3rds, and the (sub)scale of major 3rds, respectively.

Only three of the twelve hours, so far, do *not* form any oedipus subscales, or homogeneous symmetrical scales or subscales, namely, the fourth, seventh, and eleventh hours (of which more later).

Symmetrical Pentads

An exception to this simple repetition or alternation of intervals occurs with the symmetrical pentads (SPs) formed in the asymmetrical hours (i.e., those hours whose triad has an inversion).[13]

In these particular SPs, a minor interval-order is immediately reversed to become major, or vice versa. Thus SPIIm is 1+2+2+1 semitones, for example—and is *not* the same group as IIm^5, which is 1+2+1+2 semitones. SPIIM then is, of course, 2+1+1+2 semitones—a different chromatic group from SPIIm, thus found under a different heading in the chart. Whereas IIM^5 is 2+1+2+1 semitones, which is now the *inversion* of IIm^5 and thus belongs under the same heading as the latter; and if we happen to be speaking of this group in general, without needing to specify the minor or the major form, then we can simply call it II^5.

Thus it can be seen that the minor and major versions of a fundamental pattern or configuration may well form two different chromatic groups, but alternatively that one may be the inversion of the other. (The latter is always the case with oedipus groups containing an odd number of notes, thus an even number of intervals, e.g., the oedipus pentads, heptads and nonads).

Usefulness of the Tone-Clock Names

By means of the roman tone-clock names, we begin to associate all the simplest hour-group IPFs unmistakably with their respective hours, i.e., we start consciously streaming the hour-groups into the twelve hours. We do not learn to do this quickly and automatically (which is necessary) *unless* we first use the roman numerals consciously.

Moreover, the roman names also oblige us to register and to remember the structure of all the most basic configurations, since each time we use a roman name we must mentally reconstruct the configuration concerned, be it minor or major, a triad, ST, SP or whatever.

Thus the roman names teach us the twelve hours and also the most fundamental configurations—that is to say, they teach us all the most basic concepts necessary for a firm foothold in the chromatic system, giving us a simple foundation upon which we can build, a foundation from which we

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13 The designation 'SP', in the roman numeral names, is used merely to indicate the structure. In the case of the symmetrical first, sixth and ninth hours, the pentad formed is *always* symmetrical and there is always only one of them (there are no minor or major forms), thus it is not necessary to include the 'SP' in the names for the pentads in those hours.

can then branch out into the more highly differentiated groups. When we are familiar with all this, we can move without difficulty into non-hour-group IPFs such as the asymmetrical tetrads, and so on.

Larger Hour-Groups (Geminis, etc)

With the larger, more complex hour-groups, their intervallic form (which is not necessarily the intervallic *prime* form) shows each group's structure quite plainly, and fancy roman names (or any other kind of names) for every possible form of hour-group will only confuse the issue.[14] I distinguish most of these larger hour-groups only by size, by hour, by their intervallic structure, as minor and major where this applies, and as symmetrical or asymmetrical.

However, there are four distinctive symmetrical hour-group configurations or gestalts which I call the 'gemini' groups, the 'greater geminis', the 'gemini triplets', and the 'oedipus twins', respectively. A 'gemini' group consists of two identical STs (either both minor or both major) connected by pivoting—the last note of the first becomes the first note of the second. A 'greater gemini' consists of two identical SPs, minor or major, similarly connected; and a 'gemini triplet' of three identical STs, minor or major, again similarly connected. An 'oedipus twin' consists of either a minor or a major oedipus pentad followed immediately by (and likewise connected to) its inversion, which is also its opposite major or minor partner (see Figure 1.2 and the main chart in Appendix II, as well as relevant entries in the IPF chart). These verbal descriptions may sound complicated, but the configurations themselves are extremely simple and memorable. For example:

	MINOR	MAJOR
gemini	**343-343**	**434-434**
greater gemini	**1221-1221	**2112-2112
gemini triplet	**131-131-131**	**313-313-313**
oedipus twin	**1212-2121	**2121-1212

With the larger chromatic groups, which mostly do not have specific roman numeral names, there is most commonly no single distinct chromatic hour in the prime form; here the 'TC Name' and 'IPF' columns are therefore amalgamated into one column.

Usefulness of the Tone-Clock Groups/Hour-Groups

In practice, it is the smaller tone-clock groups, the smallest and/or simplest hour-groups, that are of the greatest assistance in getting to know the chromatic system. Through absorbing first the idea of twelve 'hours' or

14 It may be thought that the present roman names already confuse the issue, since the IPFs of these smaller and/or simpler tone-clock groups also show each group's structure quite plainly. I hold that the roman names are necessary to begin with, however, for the reasons already outlined, and in order to correlate the abstract structures or gestalts with the hours in which they appear.

distinctive intervallic 'climates'[15], and getting to know their associated interval patterns by way of the twelve triads, the subsequent extension of this to include all of the symmetrical tetrads and symmetrical pentads, as well as all of the oedipus groups, is then easy and virtually automatic.

Every other prime form, and every other hour-group or intervallic combination of any kind (as in music), can then instantly be analysed as a combination of these smaller characteristic gestalts which one already knows. I shall not cease to stress, moreover, that *gestalts* are the whole point, for it is well-known by now that where long-term memory and the subsconscious are concerned the gestalt—the 'chunk'—is what is stored, and not every individual element one by one. The weakness of the integer-notated pitch-class sets is precisely that they do not *show* any gestalts (nor do the tone-clock names, for that matter, but their IPFs do). This in part also explains why the chromatic groups are still largely unfamiliar to us, despite the fact that their integer notation has been around for about forty years.

The tone-clock names can be used interchangeably with the actual IPFs or (sometimes) older names, as one pleases. It is handy, for example, to be able to say simply I^8 (the first-hour octad: 1+1+1+1+1+1+1 semitones) or VI^5 (the sixth-hour pentad: 2+2+2+2 semitones), rather than specifying every element individually. The various roman tone-clock names are useful at first, as I have said, for becoming familiar with all the simplest configurations and their different appearances in the various hours. Once we know all this—so that a 2+3+3+2 for us always automatically signifies the seventh-hour minor symmetrical pentad, for example—then it can also be convenient to do away with some of the roman names and revert to the simpler intervallic forms. However, the twelve roman numerals themselves, signifying the twelve hours (represented in the first instance by the chromatic triads), are necessary initially, because without them we have only the six intervals as 'filing categories' for our subconscious mind, and this is not enough to get a proper grip on the chromatic system. If it were, we should most of us know a good deal more about it than we do, given that we have been wandering around in it for nearly a hundred years.

The fact that Schat's concept of hours can be extended to include numerous chromatic groups *larger* than the triads is extremely helpful, as well as musically logical and relevant to contemporary composition. The main hour-groups then become distinguished from the other groups, forming 'landmarks' in the chromatic territory—and such landmarks are at

[15] For myself, I add a thirteenth hour as well, namely, the 'tritone hour'. All of the other intervals can be doubled so as to form a symmetrical triad and thus have a chromatic hour of their own. Each of these intervals can be regarded as a sort of primal version of its associated hour—for instance, the first-hour triad (1+1 semitones) is 'like' the semitone, only more so. Only the tritone cannot be so doubled, and yet the tritone has an extremely distinctive intervallic 'climate' of its own. Hence I think of it as the 'thirteenth hour'—an hour which, like the twelfth, has only one member, but a very powerful one.

the outset exactly what we need. It is much too confusing, for instance, to attempt to learn all of the chromatic groups at once. If we *start* with the twelve triads and then move out into the larger tone-clock prime forms, gradually adding the other significant groups (Messiaen's modes and their chromatic complements, for instance, and Babbitt's 'all-combinatorial' hexads[16]—some of which are already also tone-clock groups) as well as the remaining tetrads, pentads, and so on, then we shall slowly be building up a real chromatic knowledge and awareness, in preference to jumping in at the deep end (and probably going under), or simply learning off lists of prime forms parrot-fashion. The chromatic groups are entities, individuals, not dead things on a page; they have their life and being in sound, in our music, and in the particular character of their properties.

IPF Column

In the fourth column (or later on, the amalgamated TC&IPF column) the intervallic prime form itself is shown.

Notation of the IPFs

In set theory, as I have said, the prime forms are shown in integer notation, their pitches (or pitch classes) being indicated by the integers [0,1,2,3,4,5... 11]. In the intervallic prime forms, by contrast, only the intervals *between* the notes are indicated, in arabics (with 1 signifying a semitone, 2 a wholetone, and so on)—just like the descriptions in earlier paragraphs, but with the plus (+) signs omitted. (We could well call this 'interval notation'.)

The notes themselves can then keep their old names if need be, with the added advantage that the IPFs are now at least a digit shorter than their corresponding set-theory definitions, and there are now only half as many actual numbers involved (representing the six interval classes, as opposed to the twelve pitch classes).

Moreover, since there is no 10 or 11, there is no need for any commas or gaps. I do often divide the intervals of an IPF into smaller sub-groups separated by dashes, however, purely in order to make them easier to remember, by emphasising inner groupings, repetitions, similarities and symmetries. Such subdivisions are not cut-and-dried: one may well prefer to subdivide the groups somewhat differently (for the same sorts of reason, perhaps, that prompt different countries to subdivide their telephone numbers differently)—in fact I often do this myself.

The triad IPFs all have a dash between the two intervals: **1-1**, for example, rather than **11**, since the latter is too easily mistaken for 'eleven', and so on (though personally I always use the roman tone-clock names for the triads, since these are unmistakable, and are a constant reminder of the hours themselves).

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[16] See Babbitt (1955).

By this, we are expressing the actual (prime-form) structure of each group in its name. This becomes clear if we consider a prime-form PC set, expressed in fixed-zero integer notation, alongside a transposition of that set: the integers change in the transposition, but the basic structure of the group does not change. In other words, the integer notation itself does not express the actual structure. The structure is represented not by the notes, but by the relationships *between* the notes, that is to say, by the intervals. This intervallic structure must be *deduced* from the integers, in integer notation. In point of fact, what we are really 'remembering', when we transpose a PC set, *is* its intervallic structure, rather than its individual notes—or at least, this is what I contend we *should* be remembering, if we want to get to know the chromatic groups.

In this connection, I have misgivings about certain set-theory operations—for example, the practice of transposing a PC set simply by adding (or subtracting) a constant number to (or from) each of its integer-elements (even though this is a supremely easy method). What one is really 'practising' in this case is not actually the transposition of a specific musical structure—it is not even a musical operation at all, but is essentially just modular arithmetic. By this method, one can happily transpose any chromatic entity whatever in a completely absent-minded fashion, without remembering or even *registering* its real (intervallic) structure at all. Such number operations will never in the world help anyone to become familiar with the chromatic system. By bypassing integer notation for the moment, the IPFs express the real structure of the prime forms and reduce the opportunity for musically-mindless arithmetic, since an IPF remains the same even when it is transposed.[17]

Criteria for Choosing a Prime Form

There are two main criteria, in Forte's theory, for determining the 'best normal order' of a chromatic group, that is to say, the note-order in which the group will appear in the prime forms table. Translated into musical terminology (and omitting a couple of finer distinctions), these criteria are as follows: the group is presented in its *most compact rising form*, with the *smallest intervals first*. The same principles operate basically with the IPFs, but two further criteria are also present.

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[17] In practice, I indicate the transposition by adding the name of the first note in square brackets after the IPF, thus: 12123 [c#] or 12123 [bb], and so on (consequently every IPF in the table could be understood as having an implied '[c]' after it). Of course, one could prefix T_1, T_2, T_3 (etc.) symbols instead, or append a subscript transposition numerator (e.g., 12123_1 or 12123_{10}), though the latter becomes awkward when a tone-clock name has a superscript numerator as well; or one can use integer notation in the brackets, instead of the old note-name, as I have done sometimes, but in general I find this too great a plethora of symbols and arabics. My experience is that the eye and mind are relieved, and understanding facilitated, by the sight of a different, familiar note-name every so often (just as they are also relieved, stimulated and orientated, I find, by the appearance of the various roman-numeral tone-clock names—provided one has grasped their principles—as visual landmarks amongst all the IPF arabics).

Symmetry

The first is that a *symmetrical* form (or permutation) of a group, where it has any, is preferred as the prime form over any *asymmetrical* form of the same group (since it is essentially simpler, having no inversion). In Forte's (and Rahn's[18]) set-theory prime forms, on the other hand, the inherent symmetry of a group is often concealed, due to the necessity (by definition) for every group to appear always in its most compact form, whereas the symmetrical form of a group is not necessarily always its most compact form. This can lead to groups which are fundamentally symmetrical being mistaken for asymmetrical groups.[19]

Smallest Number of Interval Classes

I also generally prefer any designation (provided it is still reasonably compact[20]) that identifies *fewer* interval classes in a group over some other possible designation for the same group that identifies a *larger* number of interval classes. Thus I prefer (tetrad) 144 to 431, for instance, or to 314 —which are all permutations of the same group—because the first description is simpler, in that it contains only *two* different intervals (1 and 4, a semitone and a major third), as opposed to the second description, which contains *three* different intervals (1, 3 and 4). Another possible description (permutation) of the same tetrad might be 443, which also contains only two intervals—but then I prefer 144 to 443, because the two intervals concerned are smaller.

Alternative IPFs

Where there is more than one possible symmetrical form for a group, as quite often happens, I prefer the more compact version as the main prime form. Sometimes, however, a less compact symmetrical form or forms, or even an asymmetrical oedipus form, may be listed beneath this, as an 'alternative IPF' for the same group. This may, I agree, be something of a contradiction in terms, but bear with me.

The idea of having a single 'prime form' in the first place is merely for convenience. Wherever this would be misleading, it ceases to be convenient, so I temporarily do away with it. All of these alternative IPFs (and sometimes also the 'main' IPF) are either tone-clock groups (i.e., with roman names, shown alongside them) or other hour-groups (such as geminis, etc.). Since these are the simplest and most fundamental or

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[18] Rahn (1980).

[19] Thus Kramer, in *The Time of Music* (1988), an otherwise blameless and notable study, is led to assert on p194—in connection with the 'verticals' of the first movement of Webern's Opus 29—that the pitch-class set 0158 (integer notation) is *not* 'inversionally invariant' (i.e., symmetrical). In fact it *is* (indeed it has to be: in that particular Webernian context, it cannot be anything else)—it is the symmetrical tetrad **414** (IVM^4) or alternatively **434** (XIM^4).

[20] And even when it is not, in the case of the ninth-hour groups.

distinctive intervallic forms, they are also the easiest to remember, and since memorability is the main underlying criterion for the IPFs, then these forms should appear in the IPF column. Moreover, since no one hour is in principle any more important than another, then none of these alternatives can be regarded as any more nor less important than another; if one of them goes into the IPF column, then any others that meet the same conditions should also go in. (A fuller discussion of these 'multiple-nature' groups is found under 'Other Identities'.)

The question of which is the 'correct' IPF in such a case is academic. There is no 'correct' or one-and-only prime form (there never was, in any case, since prime forms in themselves are merely a useful convention). Thus the name used will depend on how the composer cares to think of the group, or on how this group is actually disposed musically, in which case the name should merely be appropriate in the context.

Although this may seem an undesirably inconclusive approach, it has the virtue in practice of constantly reminding us that the fundamental groups in question do in fact have this multiple hour-nature—something that is certainly useful for the composer to know, and something, moreover, that is concealed by the set-theory prime forms, which give the impression not only that symmetrical groups are asymmetrical but also that everything is clearcut—all nice and tidy for our order-loving minds and examination papers.

The present approach acknowledges that things are not necessarily as simple as any convention of one-and-only prime forms is bound to make them seem—and paradoxically perhaps, it makes these particular chromatic groups not harder, but *easier* to remember, for we are accumulating more knowledge about them (in much the same way that we get to know our friends).

My main criterion, therefore, is *simplicity of structure* (rather than simplicity of *criteria*), for in my experience the simplest possible form of a chromatic group is the form in which that group is most readily stored in the subconscious, and is thus most easily remembered. At the same time, however, this is more true of the smaller chromatic groups than it is of the larger groups (though in general the smaller groups are those we learn first and those we most need to know). The most compact symmetrical forms of the six decads, for example, are *not* always the easiest versions to remember.

Via the IPFs we can also much more readily get a mental grip on the overall structure of the tetrads *as a whole*, for example, or of the pentads as a whole, and so on—and a knowledge of these more global structures of the chromatic system is a deeper-level key to remembering the chromatic groups themselves (see Appendix I, p100, for an illustration of this). The content of the larger groups is most easily remembered or worked out by their smaller complements, it is true—i.e., by the notes *omitted*—but in doing this one is not necessarily remembering any characteristic structures

of the larger groups, which are in fact quite hard to absorb. The structure of the six decads, for instance, is better grasped by treating them as a *group* (i.e., as a single six-part entity) and including their simplest *asymmetrical* permutations (see the note at the end of the decad section of the IPF chart). The 'evolutionary' implications are fascinating; for in this way, the decads as a *whole* foreshadow the singleness of the one-and-only 11-note group.

Other Identities/TC Steerings Column

Multiple-Nature Hour-Groups

Certain exceptional groups, as I have already begun to suggest, have quite a remarkable collection of possible simple forms (which, contrary to expectation, makes them more rather than less memorable). Conspicuous amongst these are the groups which consist most simply (in terms of their intervals—though least compactly, in terms of their range) of stacked fourths, i.e., sections of the 12-note scale of fourths (the ninth-hour groups). The diatonic scale and its chromatic complement the pentatonic scale are included amongst these. Here the interval-based concept of 'hours' has some interesting consequences, for such exceptional groups prove, in terms of the hours, to have a 'double nature', or a 'triple nature', or even a quadruple, quintuple, sextuple and in one case a heptuple nature.

What I mean by this is most easily illustrated by the 12-note mother-group itself, for this has a double nature that is very familiar to us—and very important it has proved. This double nature is represented by the difference between the chromatic scale (the first hour) and the scale of fourths or cycle of fifths (the ninth hour). The *pitch classes* are the same, but the *interval classes* are different—or in tone-clock terms, the *hour* is different. So simple is this difference, and yet how significant it has been. For the cycle of fifths gave rise to the entire system of traditional tonality [21]; whereas the chromatic scale, on the other hand, has given rise to music as we know it today. That is to say, the existence of this difference has been responsible, at the deepest abstract level, for the most profound qualitative *change* in the nature (not to say the history) of music—it has given birth to a new world. In a sense, Schat's 12-note 'chromatic tonalities' (see below, also p132) could be seen as a sort of extrapolation from the above, showing that the twelve notes have not only this simple first-hour/ninth-hour nature, but that they can also be rearranged, still without any note-repetitions, so as to reflect all of the other hours as well.

But for the moment we are dealing with groups of fewer than twelve notes. By way of the ninth hour, together with the fourth, seventh and eleventh hours and to a lesser extent the second, third and first hours, this multiple nature actually pervades the chromatic system. For not only the largest ninth-hour group, the full scale of fourths, possesses it: to a greater

[21] As well as to the chromatic scale itself, through the development of equal temperament.

or lesser extent every *other* ninth-hour group also possesses it. In fact roughly 14 per cent of the chromatic groups (namely 32, counting the 12-note mother-group) have a multiple hour-nature. These include most of the geminis, greater geminis and gemini triplets and many fourth- and eleventh-hour oedipus groups.

We find this 'multiple nature' in a less differentiated form in the smallest ninth-hour group, triad IX, which unlike any other triad has a minor form, a major form, *and* a symmetrical form, all three forms contained (by cyclic permutation) within the same three notes. This foreshadows the more dramatic double, triple, quadruple (IX^6), quintuple ($IX^{7, 8, 9}$) sextuple (IX^{10}) and heptuple (IX^{11}) natures that are to come in the larger groups. (Such 'foreshadowing' is a general feature of the chromatic system—in a similar way, various other properties, too, of particular larger groups are foreshadowed in a less differentiated form in nearby smaller groups.[22]) In addition to this, the fourth, seventh and eleventh hours in a number of cases share a multiple nature amongst themselves, without the ninth hour. As the size increases, groups from the third hour, then the second, and finally the first also begin to share in multiple natures with the aforesaid hours.

The main multiple-nature relationships are summarised in Figure 1.1. Various asymmetrical hour-group alternatives also exist for some of these chromatic groups, but have had to be omitted due to lack of space (they are all shown in the IPF chart, and also by hour in the Appendix II chart). Note that in three cases a chromatic group and its complement *both* have a multiple nature, namely, 4.20 and 8.20, 4.23 and 8.23, and 5.35 and 7.35 (the pentatonic and diatonic scales—see also p115).

Fig. 1.2 shows a breakdown in stave notation of the five multiple-nature hour-groups formed by the group 10.4, as an example of how to read the Fig. 1.1 diagram. (See also the double-nature STs, on p115.)

Prime Configurations?

An interesting and very unexpected feature of the multiple-nature groups is that in many cases the different hour-groups formed by a single chromatic group actually share the same fundamental pattern or configuration of intervals. This is so remarkable that one is even tempted to read into it some sort of 'intrinsic significance'. If we were in need of any 'validation' for the hour-group approach—aside from its being musically perfectly logical, viable and timely, which is quite reason enough—then this would surely be it. In one instance, this even happens four times within the same chromatic group (9.4), which forms the same configuration twice in the

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22 For example, 'triadness' is already implicit in the intervals. It is most simply foreshadowed by the tritone, which inverted at the octave forms the configuration of an 'impossible' symmetrical triad—i.e., the simplest form of triad. The real symmetrical triads then 'take over' this same configuration. (This 'evolutionary' approach may have no foundation in terms of the *actual* evolution of the chromatic system, and may well in fact be telling us more about the way our own minds work, but it is not uninteresting.)

Fig. 1.1

Multiple-Nature Hour-Groups

	I	II	IX	VII	XI	IV	III
4.23			IX^{4} (=252)	= 232			
4.26				323	= 343		
4.20					434	= 414	
5.34				2332	= 4334		
5.35			IX^{5} (=5225)	= 3223 = 2323			
5.17 [Z37]					3443	= 1441	
6.32			IX^{6} (=25225)	= 32323	= 34343		
6.26 [Z48]					43434	= 41414	
7.17 [Z37]				323-323		= 414-414	= 313-313
7.32		121221			= 343443		
7.35		212-212	= IX^{7} (=252-252)		= 343434 = 343-343		
7.22					434-434		= 131-131
8.7					3443443		= 1331331
8.20						1441441	= 3113113
8.27		121-1221 = 212-2112			= 434-4334		
8.23		1221221	= IX^{8} (=5225225)		= 4343434	= 4141414	
8.24		2112112		= 2332332			
8.14				2323-323		= 1414-414	
8.22				323-2332		= 414-1441	
9.6					3443-3443	= 1441-1441 =**144141441	
9.11		121-12121		= 323-32323			
9.12		**121-121-121		=**323-323-323 = 2332-3323			
9.4					34434434	= 14414414	= 13313313 = 31131131
9.7		21212-212			= 3443-4343		
9.9			IX^{9} (=252-25225)	= 2323-3232		= 1414-4141 = 14141414	
10.4		211211212		= 233233232	= 3443-44334 =**3443443443	= 141144141	
10.3		211212112		= 233232332			
10.1	I^{10}					= 414-414-414	= 131-131-131
10.5			IX^{10} (=252-252-252)	= 32323-3232	= 434-434-434	= 141414141	= 313-313-313
	I^{11}		= IX^{11} (=5225225225)	= 32323-32323	= 3434-434343	= 1441441441	= 1331331331
	I^{12}		= IX^{12} (=**252-252-252-252)		= 4334-434-4334	=**414-414-414-414	

Hour-Groups in the Chromatic Group 10.4

Fig. 1.2

third hour (minor and major forms), as well as once each in the fourth and eleventh hours:

13313313 (m)
31131131 (M)
14414414 (m)
34434434 (m)

This phenomenon draws our attention to what might almost be regarded as prime *configurations* of the chromatic system, which are in themselves, of course, almost completely abstract, requiring only the concept of a small interval and a larger one, together with a particular pattern of changes (alternations/repetitions).

Beyond the Octave: Giant Subscales

The ninth-hour groups and most of the larger groups shown in Figure 1.1 take us beyond the octave.

Other interesting things can happen beyond the octave as well. For example, it was stated earlier that the fourth, seventh and eleventh hours do not form any oedipus subscales. This can now be amended, for *beyond the octave* all three of these hours do form oedipus subscales. Here the third-hour subscale, **13131** (or **31313**), unexpectedly acquires a triple nature as well. Both a fourth-hour subscale, **17171** (or **71717**), and an eleventh-hour subscale, **35353** (or **53535**) can be formed using the notes of **13131**. These new oedipus subscales now have a basic range of *two* octaves, not one, before the same notes are repeated in the same order.

Also, a giant seventh-hour oedipus subscale with a range of *five* octaves can be formed from the notes of II^8, the second-hour subscale **1212121** (or **2121212**)—namely **5-10-5-10-5-10-5** (or **10-5-10-5-10-5-10**).

The above is actually the scale-of-fourths version (or 'M5 transform', as the Americans have it) of II^8 (remembering that every configuration using the semitone as the unit can be reproduced, transformed, with the perfect 4th as the unit, since whatever is true of the single-octave chromatic scale is also true—'writ large'—of the five-octave scale of fourths, as well as the seven-octave scale of fifths, not to say the scale of major 7ths, minor 9ths, etc...). This giant subscale is not included in the IPF chart, however, for to do so would open up another whole chromatic area so large that it will require a separate treatment in a later 'chromatic map'.[23]

Wherever a group has more than one simple identity, this is shown in the column headed 'Other Identities' (if it is not already shown as an

[23] There is one further subscale worth mentioning, namely the fourth-hour minor *gemini* subscale, ****141-141** (Messiaen's Mode 5).

alternative in the TC and IPF columns). In most cases, I have also included in this column the less-simple-but-most-compact version, as the one which will probably first strike the seeker-of-prime-forms as a possibility.[24]

TC Steerings (Tone-Clock Steerings)

12-Note Chromatic Tonalities

For those interested in the tone-clock theory, this same column also indicates the steering-groups available (see footnote 25 for an explanation of 'steering'), in those cases where a triad or tetrad will form a regular 12-note tone-clock 'chromatic tonality'[25]. Thus 'X^4' shown alongside triad I, for example, means that triad I can be steered by X^4 in such a way that all twelve notes are produced without note-repetitions.

In Schat's 12-note 'chromatic tonalities' there are either four triads in one hour (steered by a symmetrical tetrad in a different hour) or else three symmetrical tetrads (steered by a symmetrical triad in a different hour). The notes within each triad (respectively, tetrad) may occur in any order, and the triads (tetrads) themselves may also occur in any order—the principle of a 'collection' applied hierarchically. (See Appendix IX, p132.)

Tone-Clock Hexads

When any of four triads can come next to one another, as they can in Schat's 12-note 'chromatic tonalities', then different hexads are formed by the various pairs of adjacent triads as they move around. The hexads that

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[24] See Appendix III, pp110-12.

[25] See Schat, *op. cit.* 'Steering', a very fundamental concept, is a sort of compound transposition, closely related to Boulez's 'frequency multiplication' (though this had not occurred to Schat until I pointed it out). It concerns (successive) transpositions of the same group (but in its original *or* inverted form, or both, adjacently) onto a sequence of different notes which we can call the steering-notes (set theory would call them the 'transpositional operators'). The steering-notes together form the steering-*group*.

As pointed out above, the steering-groups shown in the TC Steerings column are almost always symmetrical, and in a different 'hour' from that of the group that is steered—the main property that gave rise to the tone-clock theory. (The same 12-note arrangements are also formed in Babbitt's 'derived sets' and Boulez's 'limited series', but there without any concept of 'hours' or of a steering-group, and without the two levels of mobility inherent in a 12-note tone-clock tonality—mobility of the groups, and mobility of the notes within each group.) Schat first published his theory in 1982 (*NRC Handelsblad*, 24 Dec). However, the earliest pure tone-clock composition, whose organisational basis is consistently and broadly the steering of one hour by another, different hour, seems to be my *Earth & Sky*, written some 14 years before this (Croft [1993]).

Steering or multiplication is really a very old idea or process, but one for which we have not formerly had a specific name. For example, in traditional harmonic theory, the entire system of keys is formed by transposing the diatonic scale, as a group, onto the successive notes of the cycle of fifths. We can now simply say that in this case the diatonic scale is *steered by* the cycle of fifths—or in terms of the present chart, that **212-212** is steered by IX^{12}: in shorthand, **212-212 /** IX^{12}—or again, more broadly, that the second hour is steered by the ninth hour. Stravinsky later extended this idea so that the *series*, in certain of his serial works, is steered by the cycle of fifths. See pp116-7 for some general examples of steering—which, in principle, can use any number of notes, and not just 12.

are thus formed (within the 12-note tone-clock steerings[26] shown in the triad section) I call the 'tone-clock hexads' (TC hexads). Twenty of the fifty hexads are tone-clock hexads.

In the hexad section, wherever the hexad concerned is one of the twenty tone-clock hexads, the (triadic) 12-note tonality or tonalities in which it occurs are shown. Here the significance of Babbitt's six 'all-combinatorial' hexads can be seen, for they occur in a large number of different 12-note triadic steerings, or 'chromatic tonalities' (see Appendix X, pp135-41).

Thus 'TC: I, II, III, VI' alongside the first hexad, I^6, means that this hexad occurs in four different (triadic) 12-note tonalities, in the first, second, third and sixth hours respectively. (We can then look up triads I, II, III and VI respectively, in the triad section, to find out the steering-tetrads, or work them out from the above, or else look them up on p132).

Similarly, alongside the group 6.2 whose IPF is 11112, you will see 'TC: III', meaning that 11112 is one of the hexads formed in at least one of the two available 12-note steerings of the third-hour triad (look it up), namely, III steered by V^4 or III steered by $VIIm^4$ (which of these two steerings it actually is you have to work out for yourself).

12-Note Dual Tonalities

Shown in smaller print in the same column are any 12-note dual tonalities (DTs) that can be formed from the hexad concerned. This refers to 12-note tonalities which contain not four triads in the same hour but two triads each in two different hours.

Thirty-eight of the fifty hexads will form dual tonalities. These include the twenty tone-clock hexads, which will naturally form dual tonalities wherever they occur in two or more regular 12-note tonalities: i.e., the given hexad can be constituted in one of the available hours, and its complementary hexad in any other available.

Thus when we see 'TC: I, II, III, VI' alongside I^6, we can automatically also take it that, in addition to the four regular 12-note tonalities already mentioned, the following dual tonalities can also be formed: I+II, I+III, I+VI, II+III, II+VI and III+VI. This means, in other words, that in this hexad *and* in its complementary hexad these six sets of triadic pairs can in both cases be formed—which can be useful to know, even if you are not interested in 12-note dual tonalities as such.

In addition to these possibilities, the entry 'DT: I+IV, II+VII' beneath this tells us that, using I^6, two further dual tonalities can also be formed, one containing two first-hour triads and two fourth-hour triads, and the other containing two second-hour triads and two seventh-hour triads. Again it is implied that the chromatic complement of I^6 (which in this case

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[26] The overall configuration of any one group steered by another group (or by itself, for that matter) is called a 'steering'. Such a configuration is of course not necessarily always a 12-note chromatic tonality, though the latter is always a steering.

is I[6] itself) will also be used, and that triads I+IV (or II+VII) are formed in both the original *and* its complement.[27]

Interval Array Column

The six-digit array shown here for each group expresses in a conveniently condensed form the total *interval content* of every chromatic group. This particular array was introduced by Martino (1961). In set theory it is known as the 'interval vector', but since the term 'vector' here has nothing whatever to do with mathematical vectors (it just sounds nicely 'scientific'), and since the merest whiff of anything mathematical is so alarming to many, I call it simply the 'interval array'—having myself introduced a similarly-based 'triad array'[28].

The six numbers in each array, reading from left to right, show in order the total number of semitones, wholetones, minor thirds, major thirds, perfect fourths and tritones contained within that group (see p118).

Z-Related Pairs

Certain groups have a 'Z' (which stands for nothing in particular[29]) followed by a number in square brackets after their PC set-number. These groups share their interval array with (i.e., have the same total interval content as) another *different* chromatic group of the same size, which is very useful for the composer to know. These particular interval arrays are underlined in the table.

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27 It is hardly surprising that this can be done when the original and its complement are the same group, but it is a little more unusual to find it can sometimes be done with two complementary hexads that are *not* the same, as for example with 6.Z3 (11121) and its complement 6.Z36 (31111), which together will form the dual tonality I+V, since triads I and V can be formed in both 11121 *and* 31111.

Some of this DT information can be found in a different format in Martino, *op cit.* His table of the 35 'source' hexads (the 15 Z-related complements—see 'Interval Content Column'—are not included) shows all of the various triadic pairs that each hexad will generate. Certain dual tonalities in my table do not appear in Martino's, due partly to the fact that he did not deal in detail with all of the Z-related complements, and partly also that he was of course not thinking in terms of the triadic mobility that is inherent in a 12-note tone-clock tonality. His table is accurate, apart from the following adjustments: '36' should be deleted from the 'One Generator' column for (his) hexad No. 14 (014578) and No. 16 (013568); '16-37' should be added in the 'Two Generators' column, for No. 31 (013689); for No. 9 (012357), '(15)' should not be bracketed; and for No. 28 (023568), '24-36' should be bracketed.

28 Gillian Whitehead, a well-established and gifted composer certainly not intimidated by complexity as such, writes: 'I found the term "interval vector" so forbidding I refused for ages to work out what it was—& I'm sure the students also' (private correspondence, 10.1.94)—an explicit example of terminology unnecessarily getting in the way of what it represents.

29 The symbol itself comes from set theory. The letter 'Z' was chosen, I suspect, simply because a capital letter was wanted and the 'Z' key is conveniently located right next to the 'Shift' key on the typewriter (and computer keyboard). Such are the explanations, sometimes, behind the most arcane and forbidding symbols.

In Forte's labels, the Z is contained within the actual PC set-number, thus 4.Z15 indicates that 4.15 shares its total interval content with another different tetrad, but does not tell us what that other tetrad is. Gillian Whitehead pointed out that a useful interpretation of 'Z' is found in the shape of the letter as a pairing symbol, e.g.:

tetrad 132 = 4.15 Z 29 = tetrad 124

and that the format 4.15 [Z29], and its reverse 4.29 [Z15], would provide a fuller identification in the chart, a suggestion I have adopted (though I have sometimes used Forte's format elsewhere).

All this means is that 132 has the same interval array as 124—namely, 111111 (interval array of the two all-interval tetrads). That is to say, both of these tetrads contain *one* semitone, *one* wholetone, *one* minor third, *one* major third, *one* perfect fourth and *one* tritone. This is the only Z-pair amongst the tetrads. There are three Z-pairs amongst the pentads and also amongst the corresponding (complementary) heptads, and fifteen Z-pairs amongst the hexads (where the two members of every Z-pair are also always chromatically complementary, as they are naturally not elsewhere).

Transpositions: Notes in Common

The interval array rather surprisingly and very conveniently also indicates at the same time the number of notes (pitch classes) that each transposition of the group *holds in common* with the original. Thus the hexad 21113, for example, whose interval array is 333321, when transposed (up *or* down) at the semitone, has three notes (i.e., a triad) in common with the original, when transposed at the wholetone has a different triad in common, and at the minor third a different triad again, at the major third a further different triad, at the perfect fourth an interval in common, and at the tritone (where the interval-array entry must always be doubled[30]: in this case from 1 to 2) a different interval in common. This is handy to know, since the degree of harmonic motion in one's music is determined by the number of notes in common from one harmonic change to the next (as well as by the rate of harmonic change).

Triad Array Column

In the next column, the 'triad array' further shows—again reading from left to right, but now in two rows—the number of chromatic triads, or the total *triadic content* contained within the group. The triad-array entries show the total number of possible chromatic triads that can be formed from the notes of group concerned, in their 'hourly' succession, thus:

I	II	III	IV	V	VI
VII	VIII	IX	X	XI	XII

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[30] See Forte, *op. cit.*, pp 30-32, for a mathematical explanation of this.

For example, a triad array reading: $\begin{matrix}133221\\312110\end{matrix}$ (the triad array for hexad 11212) means that the given chromatic group contains *one* first-hour triad, *three* second-hour triads (which may or may not be mixed, minor and major), *three* third-hour triads (likewise mixed or not—and the same applies to all the remaining minor-major hours), *two* fourth-hour triads, *two* fifth-hour triads, *one* sixth-hour triad, *three* seventh-hour triads, *one* eighth-hour triad, *two* ninth-hour triads, *one* tenth-hour triad, *one* eleventh-hour triad, and *no* twelfth-hour triads (see pp119-20).

No two chromatic groups have the same triad array. For some interesting correspondences, however, see 'Triad Arrays' in the notes for the Array Steerings chart (*Part Two*: Chromatic Map II).

Mode Subset Column (Messiaen's Modes)

The column headed 'Mode Subset' refers to the subset relationship of the group concerned to Messiaen's seven modes of limited transposition. For those who may not be familiar with them, the seven modes are as follows:

Mode 1: VI^6 **22222** (6.35)
Mode 2: II^8 **1212121** (= **2121212**) (8.28)
Mode 3: **11211211** (9.12)
Mode 4: **1113111** (8.9)
Mode 5: **11411** (6.7)
Mode 6: ****1221-1221** (8.25)
Mode 7: **1111-2-1111** (10.6)

Thus '3, 6, 7', for example, in the Mode Subset column simply means that the group concerned is a subset of Modes 3, 6 and 7.

Although this should make the analysis of Messiaen's music somewhat easier in the future, I have not done it simply in order to promote the work of my teacher. As larger chromatic groups which stand out markedly, by virtue of their exceptional properties, the modes represent some of the most striking beacons in the chromatic landscape, moreover they still have enormous potential for compositional use.

The fact that all but quite a small handful of the chromatic groups are actually subsets of one or more of the seven modes makes it useful to think of the chromatic groups also as being potentially grouped into these seven 'modal families'. The modes and modal subsets include all of the intervals, triads, tetrads and pentads, all but three hexads, all but seven heptads, fifteen of the twenty-nine octads, four of the twelve nonads, and one of the six decads—that is to say, only 37 (or 17 per cent) of the 220 charted groups are *not* modes or mode subsets. Five of these 37 are simply sections of the chromatic scale (i.e., 1st-hour groups), thus easily remembered in their own right, and another five are likewise sections of the twelve-note

scale of fourths (ninth-hour groups), so that the seven modal families, together with all the larger first-hour and ninth-hour groups account for some 88 per cent of the chromatic groups.

HC Column: 4 Hexad Classes

The column headed 'HC' (Hexad Class) is present only in the hexad section, and refers to the division of the 50 hexads into four simple classes, as follows:

CLASS 1 (6 hexads): the hexad is *symmetrical* and its chromatic complement is the same hexad (transposed). The six Class 1 hexads are Babbitt's six 'all-combinatorial hexachords'.

CLASS 2 (14 hexads): the hexad is *asymmetrical* and the chromatic complement is its inversion. Hexad 6.14(12113) is an exception: it is asymmetrical but its complement is the *same*, not the inversion.

The 15 Z-pairs:

CLASS 3 (14 hexads = 7 Z-pairs): the hexad is *symmetrical* and the chromatic complement is a *different* symmetrical hexad (or else an 'impossible' symmetrical heptad—see 'Impossible Symmetries', p29).

CLASS 4 (16 hexads = 8 Z-pairs): the hexad is *asymmetrical* and the chromatic complement is a *different* asymmetrical hexad.

Chromatic Complement Column

The meaning of the 'Chromatic Complement' column should be self-evident. The chromatic complement of a group consists of all the notes (pitch classes) that are *left* (of the twelve) after that group has been formed. These remaining notes naturally form a complementary group.

Here I have included the complementary set-numbers only in the hexad section, for everywhere else these numbers follow automatically. For example, every interval has a complementary decad (10.1 is the complement of 2.1, 10.2 is the complement of 2.2, 10.3 of 2.3, etc.), every triad has a complementary nonad (9.6 is the complement of 3.6, 9.10 is the complement of 3.10), and so on.

Note that a given group and its complementary partner always have the same number of transpositions.

Other Symbols

Double-Lined Box: Basic Interval Pattern Shared

Groups whose IPFs are contained within a double-lined box in the table have the same 'basic interval pattern' (a set-theory term)—i.e., the same number of particular interval classes in their prime forms, with the intervals differently permutated. For example, the groups 21112, 11122,

12112, 11212, 11221 and **12121** all contain three semitones and two wholetones, but in different orders. (Forte has organised his sequence of sets pretty much, though not exclusively, in accordance with their basic interval patterns, or 'bips', as the jargon has it—not to be confused with what I call interval 'configurations', in which the interval-order is fixed). Where lighter box-lines interrupt double box-lines, the 'lighter' group is foreign to the single basic interval pattern above and below it.

Bold-Type Arabics: Symmetrical Groups

Intervallic forms whose arabics are in bold type are symmetrical (when working by hand, one can simply box or underline the group, if one wishes). This includes the six intervals, for although they may appear not to be symmetrical on their own, they *are* in fact symmetrical by implication, since the two notes of any interval can always be considered as being symmetrically placed either about an invisible axis or about an absent central note.

All of the intervals, moreover, are groups of limited transposition (the first five intervals having twelve, not twenty-four, distinct forms/transpositions, and the tritone having only six). This is because none of the intervals has a real mirror inversion (the mirror inversion is the same as the retrograde), which is the general condition of symmetry.

Double Asterisk: Impossible Symmetries & The Octave

A double asterisk indicates the presence of what I call an 'impossible' symmetry, that is to say, a symmetry which is formed by adding the 'impossible' interval of an octave (in relation to the first or last note of the prime form) at the beginning or end of the group concerned.

Thus the apparently asymmetrical tetrad **115 is actually an 'impossible' symmetrical pentad, formed by adding the interval **5** at the start (****5115**—the sum of the intervals is in principle always 12), with the new first note an octave below the last note of the group (see p115).

Similarly, the apparently asymmetrical hexad **13223 is an 'impossible' symmetrical heptad, formed by adding the interval **1** at the end (****132-231**), with the new last note an octave above the first note of the group.

One can quickly determine from the IPF interval-structure whether an interval may be most easily added at the start or at the end of the group, to form an impossible symmetry. If this extra interval has already been added, so that the intervallic form shows the symmetrical structure, I use bold type, as the above examples show; otherwise I use plain type—but in either case the double asterisk is prefixed.

Some 15 per cent of the chromatic groups form such hidden 'impossible symmetries'. Moreover, in some cases a manifest symmetry will also form an impossible symmetry (for examples, see the symmetrical pentads, and also their complementary symmetrical heptads).

Recognition of the existence of the impossible symmetries as a class requires that we also admit the 'impossible interval', the octave, as being in a sense a functional interval in its own right, since without this, an impossible symmetry will remain asymmetrical, which is an incomplete description of its musical (if not its mathematical) properties. (The admission of the octave makes perfectly good sense to me, since without it we should have no chromatic space at all.) This idea will perhaps not appeal to those for whom the term 'inversionally invariant' satisfactorily describes both manifest and impossible symmetries. But structurally and musically speaking, these are two different kinds of symmetry: the first is always symmetrical in its IPF deep structure (though it will usually have some asymmetrical permutations); the second, without the added octave-note, is *asymmetrical*, but *with* the octave (hitherto a supposedly irrelevant interval in pitch-class-based deep structures) it becomes symmetrical.

Although the octave may make no mathematical difference, we can scarcely argue that there is no *musical* difference between the presence and the absence of an octave. Clearly this has made all the difference in the world to a good many composers, when for decades whole battalions devoted themselves to assiduously avoiding the octave, as the most 'taboo' of all intervals (a nice opposition to the medieval tritone taboo, the two taboos together, octave plus tritone, themselves forming the most intriguing of the impossible symmetries, the 'impossible triad'[31]).

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[31] In some respects the octave is the most interesting interval of all. Mathematically, the leap of an octave represents the process of *doubling* (as the start of a geometric progression through the successive octaves), i.e., it is a manifestation in sound of that mysterious process wherein *one* becomes *two*. It can equally well, of course, be a falling octave, in which case it represents the reciprocal process by which one becomes two, namely by division in half. Thus the octave is the musical birthpoint of two of the four fundamental mathematical processes, multiplication and division (which are also, we may note, the two means by which the simplest life-forms on earth reproduce themselves cellularly).

The oddest thing about the octave, higher or lower, is of course that we all (or most of us) recognise instantly that it is 'the same' and yet 'not the same' as its originating note. Why this should be so, nobody knows. Yet it certainly demonstrates that *number* plays a singular and fundamental role in our aural perceptions. Were it not for this extraordinary property of the octave, music as we know it would be quite impossible. The octave is thus in itself a paradox—I often refer to the octave-shift as the 'quantum leap'. All the existing musical intervals are, in the most abstract sense, contained within it, thus it represents the 'outer limits' of the entire world of music.

However, every musical interval greater than a tritone is the octave inversion of an interval smaller than a tritone. Here we find the other two fundamental mathematical processes, addition and subtraction, hand in hand. For the larger the original interval, the smaller its octave inversion, and vice versa: what is added on one side must be subtracted from the other, since the sum of the two must always add up to the octave constant. The tritone sits, in equal temperament, at the exact midpoint of the octave: it is that interval which, when inverted at the octave, always reproduces itself. All the intervals contained within the upper tritone of the octave are, by inverse symmetry, the same as those contained within the lower tritone. In point of fact, the *tritone* is the largest musical interval possible. With the tritone, moreover, from the world of the old tonality, we also have a musical birthpoint of the expansion-contraction principle (as an augmented fourth it expands, in the dominant 7th resolution, and as a diminished fifth, it contracts). And with

The Exceptions

We have already noted a few exceptions to general rules or patterns of behaviour. Throughout the chromatic system there is usually an interesting exception or two in the various behaviours and properties of different groups, and all this, when one gets to know it, gives character to the system as a whole. Here are some of the main properties with their exceptions (including those already mentioned):

Every interval can be inverted at the octave.
Exception: the tritone, which remains the same when inverted at the octave.

Chromatic triads are either symmetrical (& homogeneous–homogeneity being the simplest form of symmetry) or asymmetrical. The minor and major forms of an asymmetrical triad are not formed from the same three notes.
Exception: triad IX, which has a minor, a major *and* a symmetrical form, all three forms contained (by cyclic permutation) within the same three notes.

The chromatic hours all have members larger than triads.
Exception: the twelfth hour, whose only member is triad XII.

Minor and major symmetrical tetrads in the same hour are not contained within the same four notes, but are different chromatic groups.
Exceptions: the minor and major forms of V^4 and of $VIII^4$ are in each case contained within the same four notes (because these tetrads are also oedipus subscales).

The 24 permutations of a symmetrical tetrad introduce one new triad in a different hour from the original.

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the tritone plus its octave inversion (forming the 'impossible triad') we have the birthpoint of the reflection or mirroring process (that is, the first manifestation of symmetry), in that by octave inversion the tritone can do nothing *but* reflect itself from exactly the opposite viewpoint. Here, then, it could be said, we also have two very fundamental concepts concerning consciousness and the universe.

Indeed, the 'impossible triad' is a pleasing model for the 'great triad' of mind-matter mediated by self (or consciousness): the tritone on one side (and by implication, all that is contained within it) representing the 'world of physical reality', and that on the other side representing its reflection within the mind (or the opposite interpretation, if one prefers: physical reality as a reflection of the mind). In the world of music, both sides are present without apparent distinction between the two, but in the abstract all can be reduced to one 'original' side, with an opposite side, its inversion. Which is which cannot be determined, since the two are completely equal and opposite. The degree of paradox inherent in this model corresponds satisfyingly with the evident degree of paradox in the enigmatic processes of both the physical world and the mind. The 'distance' between 'outermost' and 'innermost' frequencies is the tritone itself: in mathematical terms, the square root of 2, an irrational number that goes on to infinity. (And in the world *beyond* the octave, there can be giant impossible triads...)

Exceptions: V^4, $VIII^4$ and X^4, whose permutations (like those of every triad) are always and only in the original hour.

Every interval can be steered by a particular hexad (and sometimes more than one) in such a way that all twelve notes are produced without any note-repetitions.
Exception: the major third cannot be steered in this way.

Every triad can be steered by a symmetrical tetrad of a different hour (and sometimes more than one) in such a way that all twelve notes are produced without any note-repetitions.
Exception: triad X cannot be steered in this way.

The aforesaid steering-tetrads are always symmetrical.
Exception: triad XII can also be steered in this way by the asymmetrical tetrads 123 and **115.

All such 12-note triadic steerings formed in an asymmetrical hour always contain two minor and two major versions of the triad.
Exception: the fourth hour, in which triad IV can be steered by X^4 in two different 12-note versions, one containing IVm four times, and the other containing IVM four times.

Any tetrad which can be steered by a (symmetrical) triad in such a way that all twelve notes are produced without note-repetitions is always symmetrical.
Exceptions: the *asymmetrical* tetrads 123 and **115 can also be steered in this way by triad XII. X^4 can also be steered in this way by the *asymmetrical* triad IVm *or* IVM.

All groups which have 12 or fewer transpositions are always symmetrical.
Exception: the asymmetrical hexad 12312, which has only 12 transpositions (6 for the original and 6 for the inversion).

Every larger chromatic group always contains within itself, at least once, its own (transposed) smaller chromatic complement.
Exception: the pentad **1221** (SP IIm) which is not contained in its own complement.

Every asymmetrical hexad has as its chromatic complement either a different asymmetrical hexad or else its own (transposed) inversion.
Exception: the asymmetrical hexad 12113 has itself, transposed but *un*inverted, as its own complement.

No group smaller than a pentad contains all six intervals.

Exceptions: the two all-interval tetrads 132 and 124 both contain all six intervals.

No group smaller than a heptad contains all twelve chromatic triads.
Exception: the all-triad hexad 11231 contains all twelve triads.

No group larger than a triad is found in all seven of Messiaen's modes.
Exception: the tetrad VIII^4 (**242=424**) occurs in all seven modes.

Every hexad contains triads in at least six different hours.
Exceptions: **13131** contains triads in only four hours: III, IV, XI and XII. VI^6 contains triads in only three hours: VI, VIII, and XII. (And for every triad formed in each of these hexads, the *remaining* triad is also always in the same hour.)

Three Laws of Chromatic Symmetry

There are three more fundamental laws of chromatic symmetry to which there are no exceptions, and which are fairly ripe with implications for the composer:

1. *Law of complementary symmetry*: every symmetrical chromatic group, of no matter what size, always has a symmetrical chromatic complement, which (if the latter should prove hard to identify without the chart) will often enough turn out to be an 'impossible' symmetry. (So far as I am aware, nobody has specifically drawn our attention to this useful fact[32].) As a corollary, every asymmetrical group always has an asymmetrical complement.

2. *Law of simultaneous symmetry at the tritone*: any group symmetrical about a given note or axis is (by rearranging the notes of the group) also symmetrical about the note or axis a tritone away, i.e., every symmetrical group has *two* points or axes of symmetry a tritone apart. As a corollary, every symmetrical group transposed at the tritone is still also symmetrical about its original centre-point or axis.

3. *Law of internal mirroring*: every possible asymmetrical permutation of a symmetrical group (and many are possible with the larger symmetrical groups) can always be inverted using *only* the (untransposed) notes of that group. This means that within any symmetrical group, one can always (by using an asymmetrical permutation of that group) form 'mini note-rows', with all their four forms (original, inversion, retrograde, and retrograde

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32 Forte gives no indication regarding the symmetry of any chromatic group larger than a hexad, i.e., he does not indicate the number of distinct forms (transpositions—12 or fewer transpositions indicating [with one exception] that the group concerned is symmetrical) for any of the larger complements, although he does indicate this for the complementary hexads. Since the number of transpositions is always the same for any group *and* its complement, one wonders why, if he was aware of this fact, he should have indicated it at all in the case of the hexad complements, when he did not do so for the larger complements.

inversion) contained within the untransposed notes of the original group.

The same applies to any asymmetrical *subset* of a symmetrical group —such a subset can also always be mirrored using only the notes of the original group.

In these respects, therefore, every symmetrical group (including the impossible symmetries, all the decads, and the 11-note group) has the same properties as the 12-note mother-group.

Consistency & Variety in Terminologies

To some, understandably, the present table of intervallic prime forms may well seem a little complicated (although to start with, one can concentrate mainly on the third/fourth columns). Those experienced in set theory will possibly be retreating hastily to their good old PC sets (and the best of luck to them), railing privately at the mixture of terminologies here, at what they probably see as a lack of consistency.

My response will be that the terminologies here are not, after all, so very inconsistent (not in musical terms anyway). And why should one now need to be so perfectly, single-mindedly consistent, for that matter, when set theory has already done all that for us, or has at least proved that it can be done (we shall overlook that naughty *doh*, fixed or moveable). One can certainly treat set theory, or any other theory, perfectly freely and constructively, taking from it or adding to it as much or as little as one pleases.

What concerns me here is the development of an appropriate, effective, economic, practical *working* terminology for the chromatic realm, a vocabulary whose terms are comprehensive, characteristic and as easily memorable as possible. It is true that the admittedly heterogeneous vocabulary presented here has more-or-less just evolved by itself over a number of years, in the directions that proved most practical, helpful and sensible to me, and that it was not the result of any preconceived master-plan. But for the purposes of really getting to know the chromatic territory, I find such a vocabulary as this actually much more effective than the completely homogeneous, essentially faceless terminology of set theory.

If memorability can be achieved somewhat at the cost of the purest, highest-level consistency, but still within acceptably consistent limits, then I for one think it a fair trade-off. Where is the advantage in perfect consistency if nobody can remember the chromatic groups? Charts may be fine for a mathematician, but for a musician they can be only a beginning, at most. If I may quite often have a number of different possible names for the same chromatic group, then so what? It is surely a characteristic of entities of any kind with which one is really familiar that one may entertain more than one name for them, despite the fact that this may not much appeal to—indeed may greatly irritate—the statisticians, registrars and

census-takers of this world. The real point is that each name describes a different property or tells us something new about the group concerned.

In fact I find a richness of names (provided this has developed organically) rather satisfying and intriguing in itself. As a fascinating example of this, there are two groups in particular, both well-known to us, which have accumulated an extraordinary variety of names (rest assured, one will not find all of the following in the IPF chart). Some of these are, just for the record:

the diatonic scale, also known as
the major scale (Pythagorean, just-intonation, and equal-tempered versions, the latter with twelve transpositions)
the natural minor scale (twelve transpositions)
the Ionian mode (plus its cyclic permutations, the Dorian, Phrygian, Lydian, Mixolydian, Aeolian and Locrian modes [names used in jazz theory also], seven authentic versions and seven plagal versions: Hypodorian, Hypophrygian, etc.)
in set theory: pitch-class set 7-35 [0,1,3,5,6,8,10]
  chromatic complement of pc set 5-35 [0,2,4,7,9]
in the present theory: symmetrical heptad: second-hour minor gemini hour-group **212-212**
  eleventh-hour minor gemini hour-group **343-343**
  'impossible' symmetrical octad ****212-2-212**
  ninth-hour symmetrical heptad IX^7(**555-555**)
  eleventh-hour oedipus heptad (**343434)
  chromatic complement of symmetrical pentad **3223** (SPVIIM)
  IIM^4 steered by a perfect fourth or fifth
  (plus a few others that would take too long to explain)

Mode 2 in Messiaen's theory, the second mode of limited transposition (with three transpositions) also referred to as:
the octatonic scale and
the symmetric scale (by various contemporary music theorists)
the diminished or diminished-seventh scale (=**2121212**), *and*
the dominant eight-note scale (=**1212121**), with three transpositions (in jazz theory)
in set theory: pc set 8-28 [0,1,3,4,6,7,9,10]
  chromatic complement of pc set 4-28 [0,3,6,9], both with three distinct forms
in Xenakis's sieve theory: the complement of sieve modulo 3 (with the tempered semitone as elementary unit of displacement), three transpositions
in the present theory: the second-hour subscale (one of the oedipus subscales)
  II^8, with a minor and a major form (**1212121=2121212**), and three transpositions
  seventh-hour giant subscale (**5-10-5-10-5-10-5=10-5-10-5-10-5-10**)
  chromatic complement of X^4
  IIm^4 (or IIM^4) steered by a tritone (plus many other subsidiary steerings)

Surely each of this enchanting array of names adds life, history, meaning and character to the group concerned. To be sure, some names (the plagal versions of the medieval modes, for instance) are now dead except for academic purposes, but each name provides a different perspective, a different way of thinking about one and the same group, and personally I would not be without any of them. Thus I am not at all disturbed by the comparatively small range of different possible names that appears in the IPF chart. Far from confusing the issue, I find these different names and identities actually help one to remember the groups concerned.

Myself, I think of the chromatic groups almost as 'persons', as individuals with different characters, histories, capabilities and propensities. This 'sociology of the tones' is so fascinating that I have cheerfully explored it for years without ever getting bored, and hope I may long continue. If we are to arrive, both as individuals and as a whole—as I believe we shall eventually, through the gradual subconscious absorption that comes with practice and perseverance—at an intuitively 'second-nature' type of chromatic knowledge or understanding, which will later also develop along the already-established, more general creative lines of 'the ideas of one generation become the instincts of the next' [D.H. Lawrence]), then I for one feel we shall need to get to know the chromatic groups consciously, as old friends. In my experience, the intervallic prime forms can provide some real help and impetus in this direction.

To know the smaller groups and their properties is to know the heart of the chromatic system: in getting to know them, one also necessarily becomes familiar, in the simplest way possible, with all the most fundamental properties and characteristics of the system itself, of which any further knowledge simply represents an increased differentiation. This is how I came to it all myself through my own explorations, at any rate, and I feel it is the most natural way—to take things step by step, at our own speed, and to move on when we feel we have satisfactorily taken in and absorbed what is presently before us.

This little book began as a handbook for my own benefit rather than anyone else's, with the aim of helping me to clarify my own thoughts and to begin to absorb the great wealth of interconnections and relationships in the chromatic system. For years I have had it at my elbow, referring to it daily, often simply browsing through it. The fairly impersonal tone here gives little hint of the excitement, enthusiasm and delight that I experienced personally, as the nature of the chromatic world dawned (and continues to dawn) on me increasingly. I should be sorry indeed if anyone were subsequently to turn its contents into some tiresome academic exercise—for alas, perhaps, it is all eminently 'teachable'. It is for composers to remember that apparently dry technical information has *musical* meaning and potential, and to give some thought to how we might make use of such

things for ourselves. If we forget this, the whole thing will indeed become a dreary affair.

Notes

CHART OF INTERVALLIC PRIME FORMS

6 DYADS

Existing Name	PC Set	TC Name	IPF	Other Identities	Interval Array	Mode Subset	Chromatic Complement
semitone	2.1(12)	1	**1**	major 7th: 11	100000	2,3,4, 5,6,7	**111-111-111**
wholetone	2.2(12)	2	**2**	minor 7th: 10	010000	all	**111-1221-111
minor 3rd	2.3(12)	3	**3**	major 6th: 9	001000	2,3,4, 6,7	**111-212-111**
major 3rd	2.4(12)	4	**4**	minor 6th: 8	000100	all	**111-2112-111
perfect 4th	2.5(12)	5	**5**	perfect 5th: 7	000010	2,3,4, 5,6,7	IX^{10} **112-111-211**
tritone sieve modulo 6 (Xenakis)*	2.6(6)	6	**6**	**6-6 *tritone subscale* thirteenth hour	000001	all	Mode 7 (Messiaen) **111-121-111**

*Note: The various 'sieves' of Xenakis's sieve theory are merely simple ways of regularly filtering or 'sifting' any form of equal-tempered musical space. In 12-note equal temperament, sieve modulo 1 is the chromatic scale itself (without any complement), sieve modulo 5 is the 5-octave (and 12-note, of course) scale of fourths (whose complement likewise covers c. 5 octaves), sieve modulo 7 is the 7-octave (12-note) scale of fifths (with a complement similarly covering c. 7 octaves), and sieve modulo 11 is the relatively gigantic (but still 12-note) scale of major sevenths (whose complement is likewise gigantic: both in fact exceed the physical limits of musical space). The remaining sieve modulos (2, 3, 4, 6, 8, 9 & 10) are all found in the present charts.

12 TRIADS

Existing Name (Section of:)	PC Set	TC Name	IPF	TC Steerings (forming 12-note tonalities)	Interval Array	Triad Array	Mode Subset	Chromatic Complemt
(chromatic scale)	3.1 (12)	I	**1-1**	X^4	210000	100000 000000	3,4,5, 6,7	**1111-1111**
(diatonic scale)	3.2	IIm (IIM)	1-2 (2-1)	$VIII^4$	111000	010000 000000	2,3,4, 6,7	2111-1111
	3.3	IIIm (IIIM)	1-3 (3-1)	V^4, $VIIm^4$ (=IXm^4)	101100	001000 000000	2,3,4, 6,7	1211-1111
	3.4	IVm (IVM)	1-4 (4-1)	VI^4, $VIII^4$, X^4	100110	000100 000000	3,4,5 6,7	1121-1111
	3.5	Vm (VM)	1-5 (5-1)	IIM^4	100011	000010 000000	2,3,4, 5,6,7	1112-1111
(wholetone scale)	3.6 (12)	VI	**2-2**	V^4, X^4	020100	000001 000000	1,3,6, 7	**2111-1112**
(pentatonic scale)	3.7	VIIm (VIIM)	2-3 (3-2)	$VIII^4$	011010	000000 100000	2,3,4, 6,7	2121-1111
"Italian sixth"	3.8	VIIIm (VIIIM)	2-4 (4-2)	$IIIm^4$, $IIIM^4$, IVM^4 (=XIM^4)	010101	000000 010000	all	2112-1111
(scale of fourths)	3.9 (12)	IX (m=M)	**5-5** (2-5) (5-2)	IIm^4, $VIII^4$, X^4	010020	000000 001000	3,4,5, 6,7	IX^9 **1112-2111**
diminished triad	3.10 (12)	X	**3-3**	------	002001	000000 000100	2,3,4, 6,7	**1211-1121**
"natural" triad minor & major	3.11	XIm (XIM)	3-4 (4-3)	VI^4, $VIII^4$	001110	000000 000010	2,3,4, 6,7	1211-2111
augmented triad sieve modulos 4 & 8 (Xenakis)	3.12 (4)	XII	**4-4**	*twelfth-hour subscale* I^4, IIM^4, V^4, $VIIm^4$(=IXm^4) X^4,123, **115	000300	000000 000001	1,3,6, 7	Mode 3 (Messiaen) **1121-1211**

Notes

29 TETRADS

Existing Name (Section of:)	PC Set	TC Name	IPF	Other Identities & TC Steerings	Interval Array	Triad Array	Mode Subset	Chromatic Complement
(chromatic scale)	4.1 (12)	I^4	**111**	steering: XII	321000	220000 000000	4,7	**111-1-111**
	4.2	-----	112	------	221100	111001 000000	3,6,7	211-1111
(harmonic minor scale)	4.3 (12)	IIm^4	**121**	------	212100	022000 000000	2,3,7	****1113-3111**
	4.4	-----	113	------	211110	101100 100000	3,4,7	121-1111
	4.5	-----	114	------	210111	100110 010000	3,4,5, 6,7	112-1111
	4.6 (12)	"SPVm" "SPVM"	**115	****5115** ("SPVM") ****1551** ("SPVm") steering: XII	210021	100020 001000	4,5,6, 7	**111-2-111**
(harmonic minor scale)	4.7 (12)	$IIIm^4$	**131**	------	201210	002200 000000	3,4,7	**311-1-113**
	4.8 (12)	IVm^4	**141**	------	200121	000220 000000	3,4,5, 6,7	****1131-1311**
	4.9 (6)	V^4 (m=M)	**151** **515**	*fifth-hour subscale* steerings: VI, XII	200022	000040 000000	2,4,5, 6,7	Mode 4 (Messiaen) **111-3-111**
(diatonic scale)	4.10 (12)	IIM^4	**212**	steering: XII	122010	020000 200000	2,7	**211-1-112**
(diatonic scale)	4.11	-----	122	------	121110	010101 100000	3,6,7	221-1111
(harmonic minor scale)	4.12	-----	213	------	112101	011000 010100	2,3,4, 6,7	121-1112
	4.13	-----	123	steering: XII	112011	010010 100100	2,4,6, 7	212-1111
	4.14	-----	214	------	111120	010100 001010	3,4,7	112-1112
	4.15 [Z29]	-----	132	all-interval tetrad	111111	001010 110000	2,3,4, 6,7	122-1111
	4.16	-----	142	------	110121	000110 011000	3,4,5, 6,7	1122-111

Notes

Existing Name (Section of:)	PC Set	TC Name	IPF	Other Identities & TC Steerings	Interval Array	Triad Array	Mode Subset	Chromatic Complement
	4.17 (12)	$IIIM^4$	**313**	------	102210	002000 000020	2,3,7	**121-1-121**
	4.18	-----	133	------	102111	001010 000110	2,3,4, 6,7	1212-111
chord of maj 7th + min 3rd	4.19	-----	144	443, 431	101310	001100 000011	3,6,7	112-1121
chord of major 7th	4.20 (12)	IVM^4 $=XIM^4$	**414** **434**	143	101220	000200 000020	3,4,7	**11-212-11**
(wholetone scale)	4.21 (12)	VI^4	**222**	------	030201	000002 020000	1,3,6, 7	****1122-2211**
	4.22	-----	223	------	021120	000001 101010	3,6,7	2212-111
(pentatonic scale) (scale of 4ths)	4.23 (12)	$VIIm^4$ $=IXm^4$ $=IX^4$	**232** **252** **555**	steering: XII	021030	000000 202000	4,7	IX^8 **1221221**
	4.24 (12)	"SP VIIIm" "SP VIIIM"	**224	****4224** ("SP VIIIM") ****2442** ("SP VIIIm")	020301	000001 020001	1,3,6, 7	**11-222-11**
"French sixth"	4.25 (6)	$VIII^4$ (m=M)	**242** **424**	*eighth-hour subscale*	020202	000000 040000	all	Mode 6 (Messiaen) ****1221-1221**
chord of minor 7th; chord of added 6th	4.26 (12)	$VIIM^4$ $=XIm^4$	**323** **343**	------	012120	000000 200020	2,3,7	****1212-2121**
chord of min 7th+ dim 5th; dominant 7th	4.27	-----	233	334	012111	000000 110110	2,3,4, 6,7	2121-211
dim 7th sieve modulos 3 & 9 (Xenakis)	4.28 (3)	X^4	**333**	*tenth-hour subscale* steerings: I, IVm *or* M, VI, IX, XII	004002	000000 000400	2,4,6, 7	Mode 2 (Messiaen) **1212121**
	4.29 [Z15]	-----	124	all-interval tetrad	<u>111111</u>	010010 010010	2,3,4, 6,7	2112-111

Notes

38 PENTADS

Existing Name (Section of:)	PC Set	TC Name	IPF	Other Identities	Interval Array	Triad Array	Mode Subset	Chromatic Complement
(chromatic scale)	5.1 (12)	I^5	**1111**	------	432100	342001 000000	7	**111-111**
	5.2	-----	1112	------	332110	231101 200000	7	211-111
	5.3	-----	1121	------	322210	123201 100000	3,7	311-111
	5.4	-----	1113	------	322111	221110 110100	4,7	121-111
	5.5	-----	1114	------	321121	220120 011010	4,7	112-111
	5.6	-----	1131	------	311221	102320 110000	3,4,7	131-111
	5.7	-----	1141	------	310132	100250 011000	4,5,6 7	113-111
	5.8 (12)	SP IIM	**2112**	------	232201	122002 020100	3,6,7	**211-112**
	5.9	-----	1122	------	231211	111112 120000	3,6,7	221-111
	5.10	II^5 (m, M) oedipus	1212	------	223111	032010 210100	2,7	111-133
	5.11	-----	2113	------	222220	112101 101020	3,7	121-112
(diatonic scale)	5.12 (12) [Z36]	SP IIm	**1221**	not contained in complement	222121	020221 200100	6,7	**311-113**
	5.13	-----	1144	2114, 4211	221311	111111 020011	3,6,7	112-112
	5.14	-----	1132	------	221131	101120 212000	4,7	122-111
	5.15 (12)	SP IVM	**4114**	1142 ****14241**	220222	100220 041000	3,4,5 6,7	**112-211**
	5.16	-----	1213	------	213211	023010 010120	2,3,7	111-331
	5.17 (12) [Z37]	SP IVm =SP XIm	**1441 3443**	1214 ****41214**	212320	022200 001021	3,7	**113-311**
(harmonic minor scale)	5.18 [Z38]	-----	1312	------	212221	012210 011110	3,4,7	111-313
	5.19	-----	1231	------	212122	011040 110110	2,4,6 7	131-113
	5.20	IV^5 (m, M) oedipus	1414	1421 (1214) 2141 (1412)	211231	010320 011020	3,4,7	311-311

Notes

Existing Name (Section of:)	PC Set	TC Name	IPF	Other Identities	Interval Array	Triad Array	Mode Subset	Chromatic Complemt
	5.21	III^5 (m, M) oedipus	1313	------	202420	003300 000031	3,7	113-131
	5.22 (12)	SP IIIm	**1331**	****31413**	202321	002220 000121	3,6,7	**131-131**
(diatonic scale)	5.23	-----	2122	------	132130	020101 302010	7	221112
(diatonic scale)	5.24	-----	1222	------	131221	010112 121010	3,6,7	222-111
	5.25	-----	2123	------	123121	020010 310120	2,7	212-112
(harmonic minor scale)	5.26	-----	2213	------	122311	011101 120111	3,6,7	121-122
chords of: min 7th/ maj 9th & maj 7th/maj 9th	5.27	XI^5 (m, M) oedipus	3434	3221(1223)	122230	010201 201030	3,7	112-122
	5.28	-----	2132	------	122212	011010 140110	2,3,4 6,7	122-112
	5.29	-----	1232	------	122131	010110 212110	4,7	212-211
	5.30	-----	1322	------	121321	001111 121011	3,6,7	122-211
dom min 9th chord	5.31	-----	1333	3332 (2333)	114112	011010 110410	2,4,6 7	121212
jazz: aug 9th chord	5.32	-----	3133	3321 (1233) 3323 (3233)	113221	002010 210130	2,3,7	121-221
(wholetone scale)	5.33 (12)	VI^5	**2222**	****22422**	040402	000003 060001	1,3,6 7	**221-122**
major 9th chord	5.34 (12)	SPVIIm =SPXIM	**2332 4334**	**2223 =****32223**	032221	000002 221120	3,6,7	**122-221**
full pentatonic scale; (scale of 4ths)	5.35 (12)	SPVIIM $=IX^5$ =SP IXM $=VII^5$ (m, M) oedipus	**3223 5555 5225** **2323	****23232** 2232	032140	000001 403020	7	full diatonic scale **212-212** IX^7
	5.36 [Z12]	-----	1123	------	<u>222121</u>	111021 101110	6,7	212-111
	5.37 (12) [Z17]	SP IIIM	**3113**	****13431**	<u>212320</u>	102200 200021	3,7	**121-121**
	5.38 [Z18]	-----	1133	------	<u>212221</u>	101210 110120	3,4,7	121-211

Notes

50 HEXADS

Existing Name (Section of)	PC Set	TC Name & IPF	Other Identities	Interval Array	Triad Array	Mode Subset	H C	Chromatic Complemt
(chromatic scale)	6.1 (12)	I^6 **11111**	all-combinatorial TC: I, II, III, VI DT: I+IV, II+VII	543210	464202 200000	------	1	same
	6.2	11112	TC : III DT: I+II, I+V, I+VIII, II+VI, II+VII, II+VIII, II+X, III+IV, VI+VII	443211	353112 220100	7	2	inversion
	6.3 [Z36]	11121	DT: I+V	433221	243321 310100	7	4	6.36 [Z3] 31111
	6.4 (12) [Z37]	**11211**	------	432321	224422 220000	3,7	3	6.37 [Z4] ****411114**
	6.5	11131	TC: VIII DT: I+III, I+V, II+IV, II+V, III+V, IV+V, IV+XI, V+X, VII+IX	422232	222350 121110	4,7	2	inversion
	6.6 (12) [Z38]	**11311**	**11511** TC : IV, V, VIII DT: I+IV, I+V, I+VIII, I+IX, IV+V, IV+IX, V+IX, VIII+IX	421242	202460 222000	4,7	3	6.38 [Z6] **41114**
Mode 5 (Messiaen)	6.7 (6)	**11411**	****141-141** *fourth-hour gemini subscale* 114-114, 14114 all-combinatorial TC: I, IV,V, IX DT: V+VIII	420243	200480 042000	4,7	1	same (Mode 5)
	6.8 (12)	**21112**	all-combinatorial **12521** TC: II, IV, VI, VII DT: I+IX, II+XI, III+VII	343230	242202 402020	------	1	same
	6.9	11122	TC : IV DT: I+VI, I+IX, II+V, II+VII, II+VIII, III+XI, V+VII, VI+IX, VII+VIII	342231	231222 322010	7	2	inversion
	6.10 [Z39]	12112 2nd hour	------	333321	134212 121120	3,7	4	6.39 [Z10] 21113
	6.11 [Z40]	11212	DT: VI+VIII	333231	133221 312110	7	4	6.40 [Z11] 32111
	6.12 [Z41]	11221	DT: III+IV, IV+XI	332232	121252 221110	6,7	4	6.41 [Z12] 23111
	6.13 (12) [Z42]	IIm^6 **12121** oedipus	**21512** TC : III, VIII DT: II+III, II+V, II+VII, II+VIII, III+V, III+VII, V+X, VII+VIII, VII+X, VIII+X	324222	044040 220220	2,7	3	6.42 [Z13] **31113**
	6.14	12113	4th hour: 41441 TC: IV DT: I+IV, II+III, II+XI, III+VII, III+XI, IV+IX, VI+XII, VII+XI	323430	124401 201041	3	2	*same* (exception)

Notes

Existing Name (Section of:)	PC Set	TC Name & IPF	Other Identities	Interval Array	Triad Array	Mode Subset	H C	Chromatic Complemt
	6.15	11213	DT: I+III, II+IV, II+XI, III+X, III+XII, IV+V, IV+VIII, VI+XI	323421	124311 120131	3,7	2	inversion
	6.16	13112	DT: I+IV, II+III, III+V, IV+VI, IV+IX, IV+XII, V+XI, VII+XI, VIII+XI	322431	113421 121031	3,7	2	inversion
	6.17 [Z43]	11231	all-triad hexad	322332	112251 121121	6,7	4	6.43 [Z17] 21311
	6.18	11321	TC: VIII DT: I+II, III+IV, IV+V, IV+VII, V+VII, V+IX, V+X, V+XI, IX+XI	322242	111350 222120	4,7	2	inversion
	6.19 [Z44]	12131	DT: III+XI, VIII+XII	313431	024420 011141	7	4	6.44 [Z19] 31311
hexatonic scale	6.20 (4)	III^6 **13131** (m=M) **31313** oedipus	**44144** all-combinatorial *3rd-hour subscale* *11th-hour subscale*: **35353=53535** *4th-hour subscale*: **17171=71717** TC: III, IV, XI, XII	303630	006600 000062	3	1	same
	6.21	21122	DT: I+VIII, II+VI, II+VIII, III+XII, IV+VIII, V+VIII, VI+VII, VI+XI, VIII+X	242412	122113 160111	3,6,7	2	inversion
	6.22	11222	DT: I+VI, II+VIII, IV+VI, IV+XII, V+VIII, VI+IX, VII+VIII, VIII+XI	241422	111223 161011	3,6,7	2	inversion
	6.23 (12) [Z45]	IIM^6 **21212** oedipus	**12421** DT: III+XI	234222	042020 440220	2,7	3	6.45 [Z23] ****321-123**
(harmonic minor scale)	6.24 [Z46]	12122	------	233331	032211 322121	7	4	6.46 [Z24] 32211
(diatonic scale)	6.25 [Z47]	12212 2nd hour	DT: V+IX	233241	030321 412130	7	4	6.47 [Z25] 23211
(diatonic scale)	6.26 (12) [Z48]	**12221** =IVM^6 **41414** oedipus =XIM^6 **43434** oedipus	**21412**	232341	020422 222040	3,7	3	6.48 [Z26] ****132-231**
(harmonic minor scale)	6.27	12123	31212 TC: V,VIII DT: II+III, II+VII, II+X, III+X, III+XI, V+VIII, VII+X, VII+XI, X+XI	225222	033020 320430	2,7	2	inversion

Notes

Existing Name (Section of:)	PC Set	TC Name & IPF	Other Identities	Interval Array	Triad Array	Mode Subset	H C	Chromatic Complemt
(harmonic minor scale)	6.28 (12) [Z49]	**12213	****312-213** ****213-312**	224322	022221 220421	6,7	3	6.49 [Z28] **13231**
	6.29 (12) [Z50]	**12321**	**21312** TC: VIII, XI DT: II+VII, II+VIII, II+X, II+XI, V+VII, V+VIII, V+X, V+XI, VII+VIII, VIII+X, X+XI	224232	022220 222420	4,7	3	6.50 [Z29] **23132**
	6.30 (12)	12312	only asymmetrical group of limited transposition TC: II, III,VII, XI DT: II+III, II+VII, II+XI, III+VII, V+VIII, V+X, VIII+X	224223	022040 240420	2,4,7	2	inversion
	6.31	12231	13122 DT: II+III, III+VI, III+VII, IV+V, IV+VII, IV+VIII, VIII+XI, IX+XI, X+XI, XI+XII	223431	013311 221141	3,7	2	inversion
(diatonic scale); (scale of 4ths)	6.32 (12)	**22122** =IX^6 **55555** =VIIM^6 **32323** oedipus =XIm^6 **34343** oedipus	9th hour: 25225 **22322** all-combinatorial TC: VI,VII, IX, XI DT: II+VII, IV+IX	143250	020202 604040	------	1	same
(diatonic scale)	6.33	21222	11th hour: 34334 TC: XI DT: II+VI, II+VII, IV+XI, V+IX, VI+VII, VII+VIII, VII+IX, VII+X, VIII+IX	143241	020112 523130	7	2	inversion
	6.34	12222	DT: II+VI, III+VI, IV+VIII, VI+VII, VII+VIII, VIII+IX, VIII+X, VIII+XI, XI+XII	142422	011113 261121	3,6,7	2	inversion
Mode 1 (Messiaen) full wholetone scale sieve modulos 2 & 10 (Xenakis)	6.35 (2)	VI^6 **22222**	all-combinatorial TC: VI,VIII, XII *6th-hour subscale* *8th-hour subscales:* **26262= 62626** **46464= 64646**	060603	0 0 0 006 0 12 0 002	3 6 7	1	same (Mode 1)
	6.36 [Z3]	11113	DT: I+V	433221	343121 111120	7	4	6.3 [Z36] 12111
	6.37 (12) [Z4]	**11114	****411-114** ****114-411**	432321	342221 021021	7	3	6.4 [Z37] **11211**

Notes

Existing Name (Section of:)	PC Set	TC Name & IPF	Other Identities	Interval Array	Triad Array	Mode Subset	H C	Chromatic Complemt
	6.38 (12) [Z6]	**41114** =IVm⁶ **14141** oedipus	11141 **55155** **24142** TC: IV, V, VIII DT: I+IV, I+V, I+VIII, I+IX, IV+V, IV+IX, V+IX, VIII+IX	421242	220460 022020	4,7	3	6.6 [Z38] **11311**
	6.39 [Z10]	21113	------	333321	232211 320121	7	4	6.10 [Z39] 12112
	6.40 [Z11]	11123	DT: VI+VIII	333231	231221 311130	7	4	6.11 [Z40] 21211
	6.41 [Z12]	11132	DT: III+IV, IV+XI	332232	221230 242110	4,7	4	6.12 [Z41] 12211
	6.42 (12) [Z13]	**31113**	**13331** 11133 TC: III, VIII DT: II+III, II+V, II+VII, II+VIII, III+V, III+VII, V+X, VII+VIII, VII+X, VIII+X	324222	222220 220420	4,7	3	6.13 [Z42] **12121**
	6.43 [Z17]	11312	------	322332	112430 141120	3,4,7	4	6.17 [Z43] 13211
	6.44 [Z19]	11313	3rd hour: 13113, 31331 DT: III+XI, VIII+XII	313431	104420 210141	3,7	4	6.19 [Z44] 13121
	6.45 (12) [Z23]	**21123	****321-123** ****123-321** DT: III+XI	234222	122022 221420	7	3	6.23 [Z45] **21212**
	6.46 [Z24]	11223	7th hour: 32332	233331	112212 321140	3,7	4	6.24 [Z46] 22121
	6.47 [Z25]	11232	DT: V+IX	233241	112121 413130	7	4	6.25 [Z47] 21221
	6.48 (12) [Z26]	**13223 **23113	****132-231** ****231-132** 11322	232341	102221 423021	7	3	6.26 [Z48] **12221**
	6.49 (12) [Z28]	**13231**	**31213**	224322	024020 240240	2,3,7	3	6.28 [Z49] **12213 = ****312-213** ****213-312**
	6.50 (12) [Z29]	**23132**	**32123** **33133** **42124** TC: VIII, XI DT: II+VII, II+VIII, II+X, II+XI, V+VII, V+VIII, V+X, V+XI, VII+VIII, VIII+X, X+XI	224232	022040 420240	2,7	3	6.29 [Z50] **12321**

Notes

38 HEPTADS

Existing Name (Section of:)	PC Set	TC Name & IPF	Other Identities	Interval Array	Triad Array	Mode Subset	Chromatic Complemt
(chromatic scale)	7.1 (12)	I^7 **111-111**	------	654321	586423 420100	------	I^5 **1111**
	7.2	111-112	------	554331	475323 422120	------	2111
	7.3	111-113	------	544431	465422 321141	------	1211
	7.4	111-121	------	544332	365352 331220	7	3111
	7.5	111-211	------	543342	344562 432110	7	4111
	7.6	111-131	------	533442	344561 132141	7	1311
	7.7	111-311	4th hour: 414114	532353	322690 253120	4,7	1411
	7.8 (12)	**211-112**	****112-4-211**	454422	364223 460221	7	SPIIM **2112**
	7.9	111-122	4th hour: 144114 (m, M)	453432	353333 362121	7	2211
	7.10	111-133	------	445332	354232 431440	7	1212
	7.11	121-112	4th hour: 141441	444441	254522 522141	------	2113
	7.12 (12) [Z36]	**311-113**	****113-2-311** does not contain complement	444342	344241 443240	7	SPIIm **1221**
	7.13	112-112	------	443532	235533 261131	3,7	1144
	7.14	IV^7(m, M) 141414 oedipus	111-221	443352	241562 433140	7	2311
	7.15 (12)	**112-211**	****211-4-112**	442443	222483 262121	6,7	SPIVM **4114**
	7.16	111-331	------	435432	245431 430441	7	1213
	7.17 (12) [Z37]	**113-311** & three geminis: 3rd-hour **313-313** 4th-hour **414-414** 7th-hour **323-323**	****311-2-113**	434541	226622 421161	3	SPIVm **1441**
	7.18 [Z38]	111-313	------	434442	234451 431251	7	1312
	7.19	131-113	------	434343	233460 352430	4,7	1231
	7.20	113-113	------	433452	214561 433141	7	4141
	7.21	113-131	------	424641	127721 221172	3	1313

Notes

Existing Name (Section of:)	PC Set	TC Name & IPF	Other Identities	Interval Array	Triad Array	Mode Subset	Chromatic Complemt
	7.22 (12)	3rd-hour gemini **131-131** 11th-hour gemini **434-434**	****131-2-131**	424542	126640 241261	3,7	SPIIIm **1331**
	7.23	211-122	7th hour: 232332	354351	242323 724150	------	2212
	7.24	111-222	------	353442	232333 563131	7	2221
	7.25	211-212 2nd hour	------	345342	144232 533440	7	3212
	7.26	121-122	------	344532	135323 361251	3,7	2213
	7.27	112-122	------	344451	134422 624151	------	3434
	7.28	122-112 2nd hour	------	344433	133253 361431	6,7	2132
	7.29	112-212	------	344352	132352 633250	7	2321
	7.30	112-221	------	343542	123533 362151	3,7	2231
	7.31	II^7 (m, M) 121212 oedipus	------	336333	055050 550550	2,7	1333
full harmonic minor scale	7.32	121-221 2nd hour	11th hour: 343443	335442	044431 432451	7	3133
	7.33 (12)	**221-122**	****122-2-221**	262623	1 2 2 226 2 12 1 122	3 6 7	VI^5 **2222**
	7.34 (12)	**122-221**	11th hour: 344334 (m,M) **322-223** **332-233** ****221-2-122**	254442	042223 663241	7	SPVIIm **2332**
full diatonic scale: (scale of 4ths)	7.35 (12)	2nd-hour gemini **212-212** (aeolian mode) IX^7 **555-555** 9th-hour gemini **252-252** 11th-hour gemini **343-343** XI^7 (m, M) **343434 oedipus	****212-2-212** (dorian mode) ****3434343** & 11th hour: 434334	254361	040423 825160	------	SPVIIM **3223** pentatonic scale IX^5
	7.36 [Z12]	111-212	------	444342	253441 542230	7	3211
	7.37 (12) [Z17]	2nd-hour gemini **121-121**	****121-4-121**	434541	146622 222161	3	SPIIIM **3113**
	7.38 [Z18]	112-121	------	434442	145451 332241	7	3311

Notes

29 OCTADS

Existing Name (Section of:)	PC Set	TC Name & IPF	Other Identities	Interval Array	Triad Array	Mode Subset	Chromatic Complemnt
(chromatic scale)	8.1 (12)	I^8 **111-1-111**	**111-5-111**	765442	6 10 8 664 6 4 2 220	------	I^4 **111**
	8.2	111-1112	------	665542	597644 662241	------	211
	8.3 (12)	**111-1113	****3111-1113** ****1113-3111**	656542	588643 641461	------	IIm^4 **121**
	8.4	1111-121	------	655552	487763 543261	------	311
	8.5	1111-211	4th hour: 141-1441	654553	466793 473241	7	411
	8.6 (12)	**111-2-111** IVm^8 **1414141** oedipus	**111-4-111**	654463	464 8 10 2 664 2 4 0	7	**115
	8.7 (12)	**311-1-113**	**113-1-311** 3rd hour: **1331331** 11th hour: **3443443**	645652	468862 442282	------	$IIIm^4$ **131**
	8.8 (12)	**1131-131	****1131-1311** ****1311-1131** 1111-311	644563	446 8 10 1 464 2 6 1	7	IVm^4 **141**
full Mode 4 (Messiaen)	8.9 (6)	**111-3-111**	1113-1113 1131-1131	644464	444 8 12 0 484 4 4 0	7	V^4 **151** **515**
	8.10 (12)	**211-1-112**	**112-3-211**	566452	486444 844460	------	IIM^4 **212**
	8.11	11111-22	4th hour: 414-4114	565552	476544 764261	------	221
	8.12	121-1112	------	556543	377463 671561	7	213
	8.13	111-1212	------	556453	376472 763560	7	321
	8.14	11-21112	4th hour: 1414-414 7th hour: 2323-322	555562	356763 844271	------	214
	8.15 [Z29]	111-1221	11th hour: 434-3443	<u>555553</u>	365673 573461	7	231 all-interval
	8.16	111-2211	------	554563	344793 674261	7	241
	8.17 (12)	**121-1-121**	**121-3-121**	546652	268842 642482	------	$IIIM^4$ **313**
	8.18	111-2121	11th hour: 434-3443	546553	267671 662571	7	133
	8.19	1121-121	------	545752	249943 462292	3	144

Notes

Existing Name (Section of:)	PC Set	TC Name & IPF	Other Identities	Interval Array	Triad Array	Mode Subset	Chromatic Complemt
	8.20 (12)	112-1-211	4th hour: 1441441 3113113	545662	248862 644282	------	IVM^4 414
	8.21 (12)	**1122-221	**1122-2211 **2211-1122	474643	3 6 4 446 6 12 3 242	7	VI^4 222
	8.22	111-2122	4th hour: 414-1441 7th hour: 323-2332	465562	264644 965271	------	322
(scale of 4ths)	8.23 (12)	22-111-22 IX^8 555-5-555 =IVM^8 4141414 oedipus =XIM^8 4343434 oedipus	2nd hour: 1221221 9th hour: 5225225	465472	2 6 2 664 10 4 6 280	------	VIIm^4 232 IX^4 555
	8.24 (12)	11-222-11	2nd hour: 2112112 7th hour: 2332332	464743	2 4 6 646 4 12 2 262	3,7	**224
full Mode 6 (Messiaen)	8.25 (6)	**1221-122 2nd hour greater geminis **1221-1221 **2112-2112	1122-112 =1122-1122	464644	2 4 4 486 4 12 2 442	7	VIII^4 242 424
	8.26 (12)	**121-1212 2nd hour oedipus twins **1212-2121 **2121-1212	2nd-hour: **212-2121	456562	166643 845481	------	VIIM^4 323
	8.27	11-21212	2nd-hour: 121-1221 212-2112 121-2112 212-1221 11th hour: 434-4334	456553	166463 773571	7	332
full Mode 2 (Messiaen) octatonic scale	8.28 (3)	II^8 1212121 (m=M) 2121212 oedipus	*second-hour subscale*	448444	088080 880880	7	X^4 333 sieve modulos 3 & 9 (Xenakis)
	8.29 [Z15]	111-2112	------	555553	356673 673451	7	421 all-interval

Notes

12 NONADS

Existing Name (Section of:)	PC Set	TC Name & IPF	Other Identities	Interval Array	Triad Array	MS	Chromatic Complemt
(chrom. scale)	9.1 (12)	I^9 **1111-1111**	****1111-4-1111**	876663	7 12 10 10 10 5 8 8 4 3 6 1	--	triad I **1-1**
	9.2	1111-1112	------	777663	6 11 10 8 8 5 10 8 4 5 8 1	--	triad IIM 2-1
	9.3	1111-1121	------	767763	5 10 11 10 8 4 8 8 3 5 10 2	--	triad IIIM 3-1
	9.4	1111-1211	3rd hour: 13313313 31131131 4th hour: 14414414 1441-1414 1411-4414 1441-4114 11th hour: 34434434	766773	5 8 10 11 10 4 8 8 5 3 10 2	--	triad IVM 4-1
	9.5	1111-2111	11th hour: 434-43434	766674	5 8 8 10 13 3 8 10 5 5 8 1	7	triad VM 5-1
	9.6 (12)	**2111-1112** two greater geminis 4th hour: **1441-1441** 11th hour: **3443-3443**	****111-222-111** 1111-1122 4th hour: 1414-4114 ****1441-4-1441**	686763	5 10 8 8 6 7 10 12 5 3 8 2	--	triad VI **2-2**
	9.7	1111-1212	2nd hour: 21212-212 11th hour: 3443-4343	677673	4 10 8 8 8 5 11 8 6 5 10 1	--	triad VIIM 3-2
	9.8	1111-2112	------	676764	4 8 8 8 10 6 8 13 4 5 8 2	7	triad VIIIM 4-2
(scale of 4ths)	9.9 (12)	**1112-2111** IX^9 **5555-5555** $=IV^9$ (m.M) 1414-1414 4th hour oedipus (above) & two oedipus twins 4th hour: **1414-4141** 7th hour: **2323-3232**	1112-1112 ****2111-2-1112** 4th hour: 1414-1441 414-41414 9th hour: 252-25225	676683	4 8 6 10 10 5 12 8 7 3 10 1	--	triad IX **5-5**
	9.10 (12)	**1211-1121** oedipus twin 11th hour: **3434-4343**	****1121-2-1211**	668664	3 10 10 6 10 3 10 10 3 8 10 1	7	triad X **3-3**
	9.11	1112-1121	2nd hour: 121-12121 7th hour: 323-32323 323-23323	667773	3 8 10 10 8 4 10 8 5 5 11 2	--	triad XIM 4-3
full Mode 3 (Messiaen)	9.12 (4)	**1121-1211** two gemini triplets 2nd hour: ****121-121-121** 7th hour: ****323-323-323**	2nd hour: 2112-1121 7th hour: 2332-3323	666963	3 6 12 12 6 6 6 12 3 3 12 3	--	triad XII **4-4** sieve modulos 4 & 8 (Xenakis)

Notes

6 DECADS

Existing Name (Section of:)	PC Set	TC Name & IPF	Other Identities	Interval Array	Triad Array	Chromatic Complemt
(chromatic scale)	10.1 (12)	I^{10} **111-111-111** gemini triplets: 3rd hour: **131-131-131** 4th hour: **414-414-414**	**111-131-111**	988884	8 14 14 14 14 6 12 12 6 6 12 2	semitone **1**
	10.2 (12)	**111-111-112	**211-1111-112** ****111-1221-111**	898884	7 14 12 12 12 8 14 14 7 6 12 2	wholetone **2**
	10.3 (12)	**111-212-111** 111-111-121	**211-111-112** 2nd hour: **2112-1-2112** 7th hour **2332-3-2332**	889884	6 14 14 12 12 6 14 12 6 8 14 2	minor third **3**
	10.4 (12)	**111-2112-11 111-111-211	****111-2112-111** ****121-1111-121** 2nd hour: 211211212 4th hour: 141144141 7th hour: 233233232 11th hour: 3443-44334 ****3443443443**	888984	6 12 14 14 12 7 12 14 6 6 14 3	major third **4**
(scale of 4ths)	10.5 (12)	**112-111-211** 111-112-111 **555-555-555** IX^{10} **141414141** IVm^{10} oedipus gemini triplets: 3rd hour: **313-313-313** 9th hour: **252-252-252** 11th hour: **434-434-434**	**121-111-121** 7th hour: 2323-32323 11th hour: 3443-43434 434-434343 344344343	888894	6 12 12 14 14 6 14 12 8 6 14 2	perfect fourth **5**
full Mode 7 (Messiaen)	10.6 (6)	**111-121-111**	****11211-11211** 11112-11112 (11121-11121) 11th hour: ****43434-43434** **3443-4-3443**	888885	6 12 12 12 16 6 12 16 6 8 12 2	tritone **6** sieve modulo 6 (Xenakis)

Note: In the case of the decads, their symmetrical IPFs are not always the easiest forms to remember. The underlined versions of the decads above are permutations which, although sometimes asymmetrical, together demonstrate a very simple overall structure for the decads, by means of which they can be remembered more easily as a whole, rather than individually:

10.1	**111-111-111**
10.2	**111-111-112
10.3	111-111-121
10.4	111-111-211
10.5	111-112-111
10.6	**111-121-111**

Part Two Chromatic Map II: Array Steerings

The interval array and the triad array in the IPF chart (see *Part One*) tell us how many, and what classes of, intervals and triads are contained within a chromatic group, but they do not tell us *which notes* of the group are used to form these intervals and triads. Nor does the triad array tell us how many minor and major triads there are, in a given hour, for any particular chromatic group—it simply tells us the overall number of triads for each hour. We could work all this out for ourselves, if necessary, given the actual arrays (or even without them, for that matter), and we might even prefer to. The Array Steerings chart will save us this trouble if we wish, however, or will provide a double-check, for it tells us exactly *which transpositions* of the various intervals and triads occur within any group, and also which of these triads are minor and which are major.

The array steerings tell us, in other words, what the *steering-groups* are for the intervals and triads contained within each chromatic group.

HOW TO READ THE AS CHART

The intervals and triads themselves (as the groups being steered) are shown in the columns at either end of each page. Thus, reading across the page, the top line beneath the headings always refers to semitones, the second line to wholetones, the third line to minor thirds, and so on down the page, running through all the intervals and triads (and the same applies to every page).

Reading across the headings at the top, page by page, we find all the chromatic groups (in the order in which they appear in the IPF chart) from the tetrads through to the decads. Reading down any one column, we find all the various steering-notes (transpositional operators) and/or steering-groups of the various intervals and triads contained in the particular group named at the top of that column.

Arabics *not* enclosed in square brackets indicate intervals. Roman numerals indicate tone-clock hour-groups (a roman numeral without any superscript number always indicating a triad). Minor and major hour-groups have an 'm' or 'M', respectively, after their roman numeral. 'SP' prefixed indicates a symmetrical pentad (used only with the asymmetrical hours). Bold type indicates a symmetrical group and double asterisks indicate an 'impossible symmetry'. (See the IPF notes for a full discussion of all these terms.)

E.g., for the first group, I^4, the roman numeral I alongside the arabic **1** in the end column means that the interval of a semitone (**1**) occurs in the group I^4 *steered by* triad I—in other words, there are three different semitones in I^4; each of these three semitones has a different lowest note (a

steering-note, or transpositional operator); these three lowest notes, or steering-notes, *together* form a steering-*group*, which in this case happens to be triad I.

Arabics in square brackets after a steering-group tell us *which note* of the original IPF the steering-group *begins* on. If there is *no* number shown in square brackets, this means that the steering-group named starts on the *first* note of the original IPF (the bracketed number '[0]' has here been omitted, in the interests of chart clarity).

In the above example, for instance, there is no such number alongside the steering-group, triad I, therefore this steering-group starts on the first note of I^4, the original IPF. A '[1]' alongside the steering-group would tell us that this steering-group starts on the note *a semitone above* the first note of the original IPF; a '[2]' tells us that the steering-group starts on the note a *wholetone* above the first note of the IPF, a '[3]' that it starts on the note a *minor third* above the first note, and so on—a form of (moveable-*zero*) integer notation that can apply regardless of which transposition of the original IPF we are using.

If there is no steering-group named at all, but *only* a number in square brackets, this tells us that the interval or triad concerned (refer across to the end columns to find out what it is) occurs only *once* in the IPF whose column this is, and that this interval or triad therefore has no steering-*group*, but has only a steering-*note*—which is the note shown inside the square brackets.

At this point, the best way to get to know the chart is to select any chromatic group at random, look up its interval and triad arrays in the IPF chart, then work out for yourself what intervals and triads are actually contained in this group—and then compare your findings with what is shown for that group in its column in the Array Steerings chart. Do this for one or two different groups and you should have a perfectly adequate grasp of how the AS chart works.

Figure 2.1 overleaf shows the steering-entries for the group 9.12 (Messiaen's Mode 3) written out in stave notation, as an illustration of how to translate the chart symbols. (Note that Mode 3 is virtually a 'mini chromatic system' in itself, with its regular steerings of all twelve chromatic triads, including all minor and major forms.) See also pp118-20.

Further Ramifications of the Array Steerings

Do the array steerings need to stop at the intervals and triads? Clearly, we could go on to construct further 'tetrad arrays', 'pentad arrays', and so on, along with all their steering-groups. But these would be far more lengthy and much harder to follow (since there are so many more possible tetrads and pentads than there are intervals and triads) and would thus add considerably to the complexity of the various group networks without, I think, adding very greatly to their substance or character. For the larger any subset of a given group is, the fewer the number of times it can

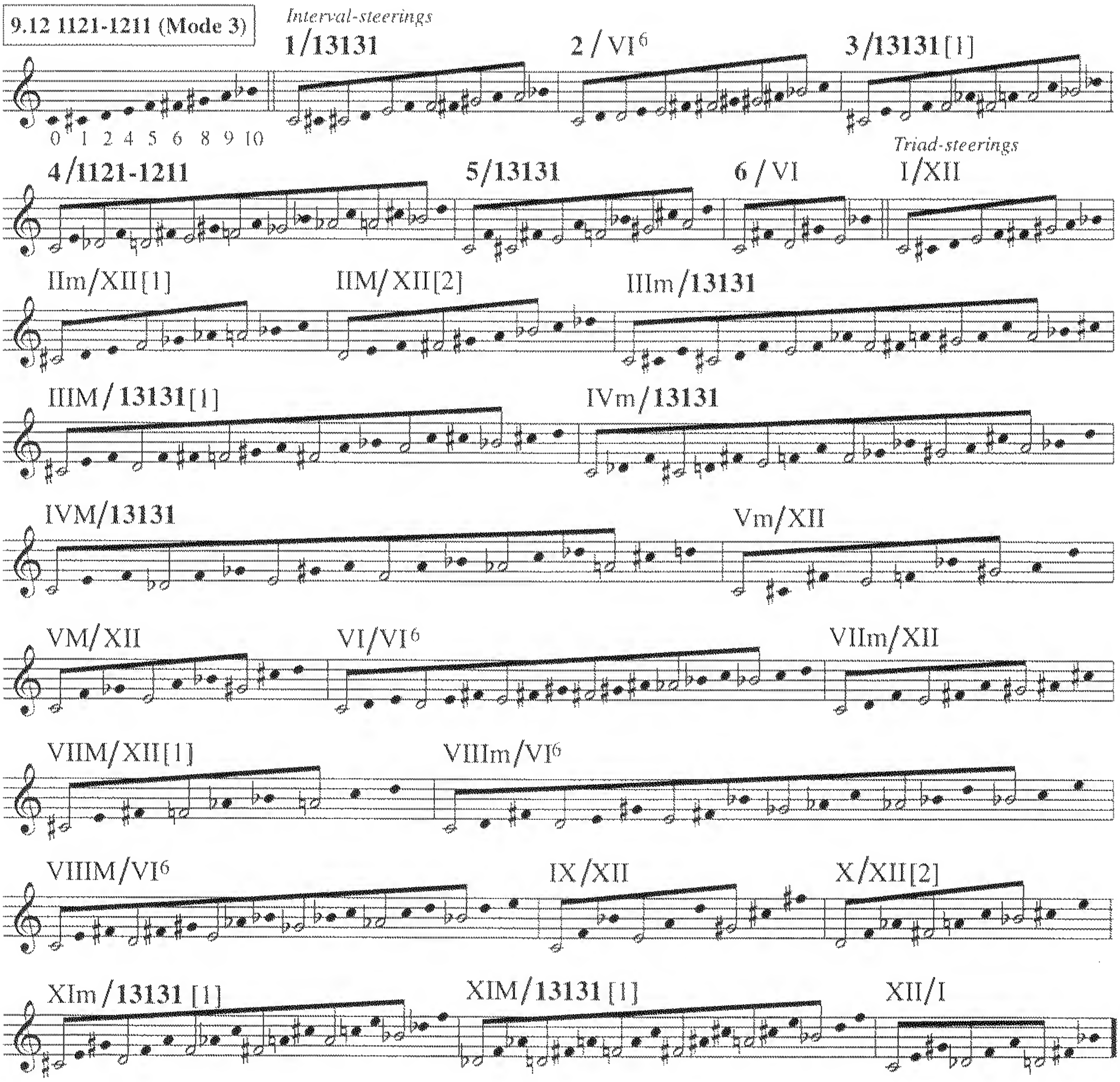
9.12 1121-1211 (Mode 3)
Interval-steerings
1/13131
2 / VI^{6}
3/13131[1]
0 1 2 4 5 6 8 9 10
4/1121-1211
5/13131
6 / VI
Triad-steerings
I/XII
IIm/XII[1]
IIM/XII[2]
IIIm/13131
IIIM/13131[1]
IVm/13131
IVM/13131
Vm/XII
VM/XII
VI/VI^{6}
VIIm/XII
VIIM/XII[1]
VIIIm/VI^{6}
VIIIM/VI^{6}
IX/XII
X/XII[2]
XIm/13131 [1]
XIM/13131 [1]
XII/I

Fig. 2.1

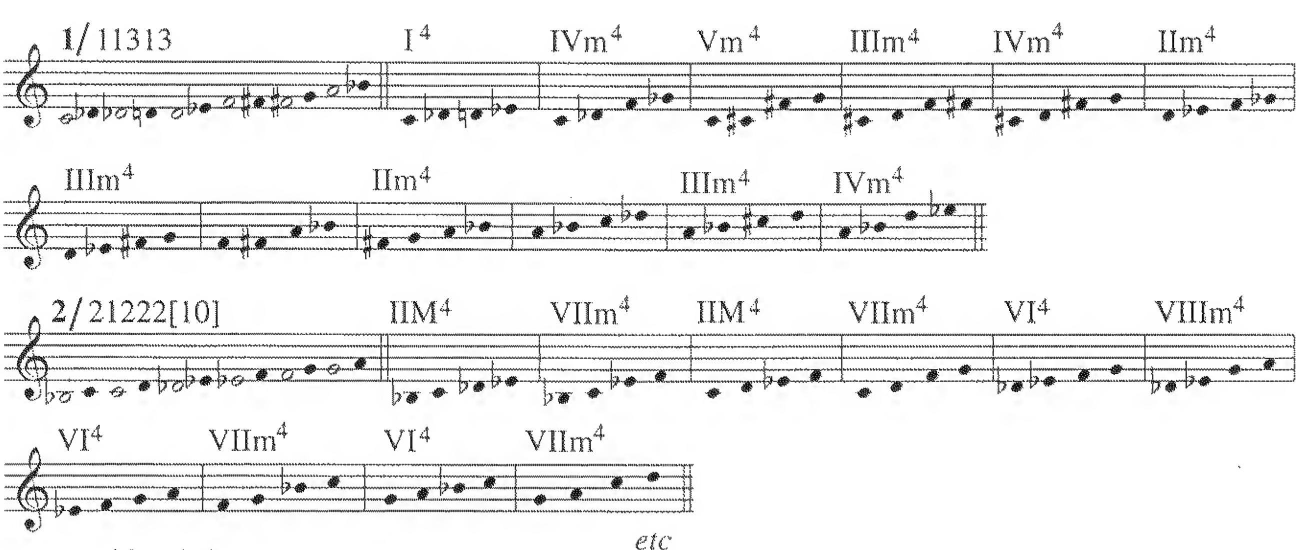
1/ 11313
I^{4}
IVm^{4}
Vm^{4}
IIIm^{4}
IVm^{4}
IIm^{4}
IIIm^{4}
IIm^{4}
IIIm^{4}
IVm^{4}
2/ 21222[10]
IIM^{4}
VIIm^{4}
IIM^{4}
VIIm^{4}
VI^{4}
VIIIm^{4}
VI^{4}
VIIm^{4}
VI^{4}
VIIm^{4}
etc

Fig. 2.2

generally be expected to occur within that group. In that case, such a subset will probably already be shown amongst the present array steering-groups (which are, of course, also subsets of their original IPF), or will be nested as a subset of one of these subsets, as we shall now discuss.

Reverse Steerings & Nested Subsets

More can be read, for instance, from the present array steerings than simply the total interval and triad content of the particular chromatic group concerned. As well as showing this, they also show every largest subset that occurs twice or three times within the main group.

For example, we may read, for the group 9.11, that the semitone occurs six times within this group, steered by 11313. But this *also* means (through what I call the 'steering-partner' relationship—of which this is but one example, amongst other possible kinds) that the group 11313 occurs twice within the group 9.11, steered by a semitone. That is to say, every steering-relationship shown in the array steerings (or any steering-relationship at all, for that matter) can also be *reversed*. This in mathematics would merely be the same as saying that x times y is equal to y times x, but where pitches are concerned, although the end result (the total notes produced) is the same, there is musically a real difference of grouping and sound between a semitone steered by a hexad (which produces six different semitones) and a hexad steered by a semitone (which produces two hexad transpositions a semitone apart). If one wanted to make these two structures audible, for instance, they would call for rather different musical treatment—in other words, the difference between the two is potent for exploitation by the composer (and things we can exploit are the things we are looking for). To my own ear, there is something rather fascinating about the aural difference between a steering and its reverse steering; they sit curiously appropriately in relation to each other.

The fact that all the array steerings can be reversed is a point worth stressing, for if it might fairly legitimately be objected that in one sense the array steerings more-or-less reduce every chromatic group to different versions of the same thing, then their reverse steerings do just the opposite, that is, they show exactly which groups *in addition* to the intervals and triads occur most frequently in the chromatic group concerned, thus characterising each group in terms of its repeated subsets in a quite specific way.

But is this really so? We may be browsing through Forte[1], for example, where we might come upon some such snippet as this, on p171: 'pc set 4-19 [i.e., tetrad 144]...is contained in 9-4 nine times—more than any other tetrachord [tetrad]'. So we look up the group 9.4 in the array steerings, to see whether the fact of 144's prominence is actually reflected there, and lo, it does not appear at all. Why not?

[1] Forte (1973).

The reason is that if (to return for a moment to our earlier example) the subset 11313 occurs twice, steered by a semitone, within the main group, then naturally any of 11313's possible subsets, as well as the entire network of 11313's array steerings, *also* occur twice steered by a semitone, within that same main group.

So now we go back to the 9.4 column, and look for the next largest subsets. We find that three heptads (111-131, 11-3131 and 113-113, steered by a semitone, a major third and a perfect fourth respectively) each occur twice, and after checking the array steerings for *these* three groups (which are 7.6, 7.21 and 7.20 respectively), we see that in the group 11-3131, 144 is by far the most prominent steering-group, and that it also appears in 7.6 and 7.20. We could then, if necessary, figure out (from the given starting-notes) exactly what its total transpositions are in 9.4.

Each column, therefore, shows us the *largest* saturation subsets for the IPF concerned. Its *smaller* saturation subsets (and also the IPF's various sub-networks) will be found by looking up, in turn, the columns for these larger saturation subsets. Thus from the interval- and triad-array steerings alone a good deal of further information can be gathered.

The interval steering-groups can also easily be recombined, moreover, so as to form all the symmetrical tetrads (always hour-groups) possible in the particular main group. Figure 2.2 on page 70 shows (for the group 9.11) how this can be done.

The array steerings show, therefore, that from any chromatic group a characteristic 'natural' steering network can be generated, based upon the intervals and the chromatic triads it contains. Of these networks no two are exactly the same, though there are often strong resemblances, where the originating groups have much of their interval structure in common.

Predominant Steering-Groups

Considering the chart as a whole, certain array steering-groups predominate quite conspicuously over the rest, in terms of their frequency of appearance (moreover, certain other groups do not appear at all as steering-groups). Since the array steering-groups may be thought of as existing in one sense at a 'deeper level' within the chromatic system (being those groups which *steer* the smallest, most fundamental groups, the intervals and triads; and a steering-group always being at least one level deeper than the group that it steers), there is some argument for regarding these predominant steering-groups as 'more significant' than the other chromatic groups.

Of the tetrads which appear as array steering-groups, for example, 144, I^4 and X^4 occur much more often than any of the other tetrads do, with V^4, 112 and 223 being the next most frequent, and $VIII^4$ not appearing at all (by contrast, $VIII^4$ is the only tetrad and the largest group to occur in all seven of Messiaen's modes). Notably, 144 appears nine times as a steering for the major third amongst the heptads, also six times

as a steering-group for the major third amongst the hexads, and six times each as a steering-group in 7.21, 8.17, and 8.20.

The asymmetrical tetrads appear much more often as steering-groups amongst the nonads than they do elsewhere (the two all-interval tetrads, the Z-pair 132 and 124, both occur as steering-groups in 9.2, 9.3, 9.7, 9.8 and 9.11, and occur *twice* in 9.5). Likewise of the pentads, 1333, 1141 and I^5 are by far the most frequent (1333 appearing no less than twelve times amongst the nonads, as a steering-group for triad X), followed by **3223** (the pentatonic scale), 1313, **3113** and 1112.

On the other hand 1212, **4114**, 1213, 2123, 2132, 3133 and VI^5 do not appear at all as steering-groups. Similar absences and frequencies of appearance can be observed also amongst the larger steering-groups, reaching their most notable point with the nonads, where, of the twelve that are possible, only the five symmetrical nonads (the complements of the five symmetrical triads, I, VI, IX, X and XII) ever appear as array steerings.

The above are all examples of the ways in which a homogeneous system—as the chromatic system, based upon equal-tempered semitones, is essentially homogeneous—tends typically to 'coalesce' at certain 'attractive gravitational' points (according to the native properties of its own inherent, inner sub-group combinations), and to develop its own characteristic pathways or 'cheodes', its own 'inner landscape', as it were; in this case, a chromatic landscape whose specific contours we are tracing in some detail in the present charts.

A further 'weighting' can also be observed, brought about by the starting-notes (indicated by the notes in square brackets) of the various steering-groups for any one main group. For example, the group I^4 contains four different notes—in integer notation [0,1,2,3]—but only one of these notes, namely [0], ever appears as a starting-note; the next tetrad, 112, contains the notes [0,1,2,4], but only two of these notes appear as starting-notes—[0] six times and [1] twice—and so on. I am not suggesting that such apparent hierarchies ought necessarily to be pursued, but rather that, if one happened to be seeking some basis for a note-hierachy within any particular chromatic group, then this 'natural' weighting[2] amongst the starting-notes might well serve the purpose.

Array Networks of The Modes & Subscales

The networks for Messiaen's modes and their chromatic complements (which themselves include the whole system of symmetrical [homogeneous]

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[2] Such weighting is of course only 'natural' because we have chosen to take the *lowest* note of a group as the steering-note. Steering or multiplication can in principle also operate using any *other* note of a group as the steering-note, a refinement we shall not enter into here. (Schat began, for example, by using the *highest* note as the steering-note, but has since changed to the lowest note, with some encouragement from me.) Indeed, the steering principle itself is merely a useful hypothesis (useful for composers, that is), with quite some foundation in past musical history, it is true, but with no 'absolute' implications.

and oedipus subscales) stand out markedly from the rest, by virtue of their exceptionally homogeneous steering-groups. The modes and their (charted) complements are as follows:

4.9 V^4 **151** (fifth-hour subscale, complement of Mode 4)
4.25 VIII^4 **242** (eighth-hour subscale, complement of Mode 6)
4.28 X^4 **333** (tenth-hour subscale, complement of Mode 2)
6.7 Mode 5: **11411** (complement is itself)
6.20 **13131**: third-hour subscale (skeletal in Mode 3; complement is itself)
6.35 Mode 1: VI^6 **22222** (sixth-hour subscale, complement is itself)
8.9 Mode 4: **111-3-111** (complement of V^4)
8.25 Mode 6: ****1221-1221** (complement of VIII^4)
8.28 Mode 2: II^8 **1212121** (second-hour subscale, complement of X^4)
9.12 Mode 3: **1121-1211** (complement of triad XII=twelfth-hour subscale)
10.6 Mode 7 (complement of tritone=tritone subscale)

The inter-relationships that exist *between* the various steering-networks for the above groups are interesting. From the array steerings, we see, for example, that every steering-group for V^4 and VIII^4 consists of a tritone, which is the chromatic complement of Mode 7. The steering-groups for Mode 5 consist only of V^4 (complement of Mode 4) and a tritone. The steering-groups for **13131** (the third-hour subscale) consist only of triad XII (complement of Mode 3), of **13131** itself, and a wholetone. The steering-groups for VI^6 are VI^6 itself and a wholetone. The steering-groups for Mode 4 consist of a tritone, I^4, V^4, X^4 (complement of Mode 2) and Mode 5. Those for Mode 6 are a wholetone, a tritone, V^4 and X^4, SPIIm (**1221**), and Mode 1; those for Mode 2 are X^4 (its own complement), IIm^4 (**121**) and Mode 2 itself; those for Mode 3 are triads I and VI, triad XII (Mode 3's own complement), **13131** and VI^6; and those for Mode 7 are a wholetone, I^5, Modes 2, 4, 5 and 6, and 12312 plus its inversion (a significant tone-clock halfhour or hexad, also the one and only asymmetrical group of limited transposition). Thus there is a fairly remarkable deeper network interconnecting the steering-groups for these groups with the groups themselves, a set of relationships that certainly marks out this whole smaller system of modes and symmetrical subscales quite distinctively from the remaining chromatic groups.

'Mini Tone-Clock-Type' Harmonic Fields

The symmetrical structure of every mode, moreover, is such that *any* chromatic group which can be found in a given transposition of a particular mode (i.e., any mode subset) will *also* always occur there *steered by* a particular interval or larger group (which is always one of the steering-groups shown above for that mode). That is to say, the group concerned

will always occur at least *twice* within that one transposition of the mode (and if asymmetrical, its inversion will occur at least twice again.) Thus it is axiomatic that any group which occurs in (a given transposition of) Mode 2, for example, *also* always occurs there steered by X^4 (at least); likewise any group occurring in Mode 3 can always be steered there (at least) by triad XII (see Fig. 2.1); any group occurring in Modes 1, 4, 5, 6 or 7 can always be steered there at least by a tritone.

This means that for any such mode subset, a sort of 'mini tone-clock-type' steering-network, or harmonic field, can be set up (containing 6, 8, 9 or 10 notes, depending on the mode, rather than 12 notes). This steering-network naturally has as many transpositions as the mode itself does. In most cases such a network will contain note-repetitions, it is true, but that need not necessarily be a problem; it can be just as much an advantage, since it will provide pivot notes-in-common between harmonic changes. (When the group concerned is modally complementary to itself, this can also be done without any note-repetitions within a given transposition of the mode.) Moreover, if any chromatic group is a subset of *more* than one of the modes, then more such mini-networks can similarly be set up for that same group, one for each of the modes in which it is a subset. If we recall that well over 80 per cent of the 222 chromatic groups are subsets of one or more of Messiaen's modes, then it becomes clear that this is potentially a powerful set of chromatic properties, capable of a very wide application indeed.

Symmetrical Groups: Mirror-Steerings

For every chromatic group that is symmetrical (including the impossible symmetries), each of its interval steering-groups is always symmetrical, and its triad steering-groups are also either symmetrical in themselves, or symmetrical by a combination of minor and major triads within any one hour. Thus for I^9, we see, for example, that triad IVm occurs five times, steered by 1114, which is an asymmetrical pentad, but that the inversion, triad IVM, *also* occurs five times, steered by the *inversion*, 4111. Such mirror-steerings occur without exception, for all of the symmetrical chromatic groups.

Asymmetrical Groups: The Inversion Network

For the remaining chromatic groups, which are asymmetrical, a larger symmetrical network is obtained by including the array steerings of the *inversion* of the main group. These are not shown in the chart, but can easily enough be worked out from them, since they invariably involve mirror-steerings similar to those above. Thus the original IPF form and the inversion of the asymmetrical octad 8.5, for example:

	8.5	
	1111-211	112-1111
1	11131	13111
2	1122	2211
3	IIm⁴	IIm⁴ [1]
4	1144 [2]	4411 [4]
5	1141 [1]	1411 [7]
6	I	I
I	114	411
IIm	IIm	IIIM [1]
IIM	IIIm	IIM [2]
IIIm	IIM	IIIm
IIIM	IIIM	IIm [1]
IVm	114 [1]	VM [7]
IVM	Vm [2]	411 [8]
Vm	1141	Vm⁴
VM	Vm⁴ [7]	1411[7]
VI	VI	VI
VIIm	**1** [1]	**2**
VIIM	**2** [1]	**1** [1]
VIIIm	114	VIIIm
VIIIM	VIIIM [8]	411 [8]
IX(**5-5**)	Vm [2]	VM [2]
X	**1**	**1** [1]
XIm	**1**	**4** [1]
XIM	**4** [8]	**1**
XII	[0]	[0]

Fig. 2.3

From this typical example, it will be seen that every invertible steering-group, and every other relationship that can possibly be inverted, becomes so, for the inversion of 8.5 shown on the right. That is, the steering entry for every minor triad on the left becomes, inverted, the entry for the corresponding major triad on the right, and vice versa. Thus the total network for the group becomes symmetrical, in quite an interesting and satisfying way. (See pp119-20 for a further example in stave notation.)

Array-Networks of the Z-Pairs

Interval Arrays

I have said that no two networks in the array steerings are completely identical. We know, however, that certain pairs of chromatic groups (the so-called Z-related pairs) have the same interval array, that is, they both contain the same number of semitones, wholetones, minor thirds, major thirds, etc., so that the Z-pairs are to this extent the same.

The array steerings show us that this apparent sameness can vary somewhat, however, for although each group in a Z-related pair may contain, for example, four semitones, the actual *number* of intervals involved may be less significant than the respective *steering*-groups of those intervals. Four semitones steered by X^4, for example, are a very different matter from four semitones steered by I^4: in the first case eight notes (forming Mode 2) are involved, and in the second case only five notes (forming I^5). Obviously the two products are very different: the mutual 'fourness' is here over-ridden to a considerable extent by the difference between the two steering-groups. (This difference can also be compositionally useful, for variety, however.)

In most cases there proves to be less in common than one might imagine where the apparent identity of the Z-pair interval arrays is concerned. The array steerings show us exactly what the steering differences are, something that is not demonstrated by the interval arrays alone. From the array steerings, we discover that, of all the Z-related pairs of chromatic groups, only a handful also contains a maximum of steering-group correspondences (though these always occur in a different order). As an example of this maximum correspondence, consider the entries for the partner pairs, 6.6 Z 38, and 6.19 Z 44:

Interval Array	421242 Z		313431 Z	
	6.6 **11311**	6.38 **14141**	6.19 12131	6.44 31311
1	IVm^4	**115 [10]	XIm	IVM[3]
2	**5**	**1**[10]	[1]	[7]
3	[2]	[0]	IIIm	XIm [9]
4	**1** [1]	**5** [1]	144 [3]	144 [3]
5	**115	IVm^4	IVM	IIIm [3]
6	**1**	**5**	[1]	[3]

Fig. 2.4

The following other Z-related pairs have a similar, or near-to-similar, correspondence: 5.12 Z 36; 5.17 Z 37; 5.18 Z 38; 7.17 Z 37. Apart from these Z-pairs, the various interval steering-groups of the remaining Z-pairs (by far the majority) bear little resemblance to one another.

Triad Arrays: 'Triangular' Relationships

The triad arrays of the chromatic groups, on the other hand, are never actually identical to one another, as we have observed, but the question arises as to whether any of the Z-pairs exhibit any interesting correspondences in their triad arrays and steering-groups. For this purpose, we shall forget the actual starting-notes.

Here, in most cases, of the nineteen triad array-entries in the column, at least five correspond, often six or seven, and sometimes as many as nine, but this is still less than half. In just a few cases, however, there is some sort of correspondence, whether literal or by an interchange of entries, between *all* of the entries. Of these pairs, the most interesting are the following three:

Triad Arrays	202460 222000	220460 022020	226622 421161	146622 222161	365673 573461	356673 673451
	6.6 **11311**	6.38 **14141**	7.17 **113-311**	7.37 **121-121**	8.15 111-1221	8.29 111-2112
I	**5**	**1**	**4**	[ ]	I	IVm
IIm	—	[ ]	[ ]	**4**	IIm	VIIIm
IIM	—	[ ]	[ ]	**4**	Vm	**3**
IIIm	[ ]	—	IVm	IIIM	VIIIM	IIIm
IIIM	[ ]	—	IVM	IIIm	**3**	VM
IVm	**1**	**5**	IIIm	XIm	IX	I
IVM	**1**	**5**	IIIM	XIM	IVM	IVM
Vm	Vm	Vm	[ ]	[ ]	421	Vm
VM	VM	VM	[ ]	[ ]	VM	231
VI	—	—	**2**	**2**	VI	VI
VIIm	[ ]	—	**4**	[ ]	**3**	VM
VIIM	[ ]	—	**4**	[ ]	VIIIM	VIIM
VIIIm	[ ]	[ ]	[ ]	[ ]	VIIIm	124
VIIIM	[ ]	[ ]	[ ]	[ ]	132	VIIIM
IX(**5-5**)	**5**	**1**	[ ]	**4**	IVm	IX
X	—	—	[ ]	[ ]	X^4	X^4
XIm	—	[ ]	XIM	IVm	XIm	VIIIm
XIM	—	[ ]	XIm	IVM	Vm	**3**
XII	—	—	[ ]	[ ]	[ ]	[ ]

— = a nil entry
[ ] = one appearance only of the triad concerned

Fig. 2.5

With 6.6 Z 38, seven pairs of array-entries correspond literally, another four correspond by interchange of '**1**' and '**5**', and another eight by interchange of '—' and '[ ]'. With 7.17 Z 37, seven pairs of array-entries correspond literally, another six correspond by interchange of '**4**' and '[ ]', and in the remaining six there is an interesting triangular relationship between the third, fourth and eleventh hours: III is steered by IV and III, IV is steered by III and XI, and XI is steered by XI and IV.

The most remarkably intricate correspondences occur with the last pair, 8.15 Z 29 (the complements of the two all-interval tetrads, 132 and 124, respectively). Here three pairs of array-entries correspond literally (X steered by X^4, and IVM and VI steered by themselves). Another four pairs, involving the fifth and eighth hours, correspond by hour and inversion: VM is steered by itself and Vm by itself, VIIIm by itself and VIIIM by itself. Four more pairs correspond by interchange of 'V' and '**3**'.

Then there are three sets of more-or-less triangular relationships, first between the first, fourth and ninth hours (I is steered by itself and IX is steered by itself; IX and I are both steered by IVm; and IVm is steered by IX and by I). There is a second quasi-triangular relationship between the second, third and eighth hours (IIm is steered by itself and by VIIIm; IIIm is steered by VIIIM and by itself). A third quasi-triangular relationship exists between the seventh, eighth and eleventh hours (VIIM is steered by VIIIM and by itself, and XIm is steered by itself and by VIIIm). And finally there are four pairs of entries (again involving the fifth and eighth hours) which contain the two all-interval tetrads (the chromatic complements): Vm steered by 421 is balanced by VM steered by 231; and VIIIm steered by 124 is balanced by VIIIM steered by 132. This is surely a fairly astonishing set of correspondences. The triad arrays themselves, not surprisingly, also correspond closely:

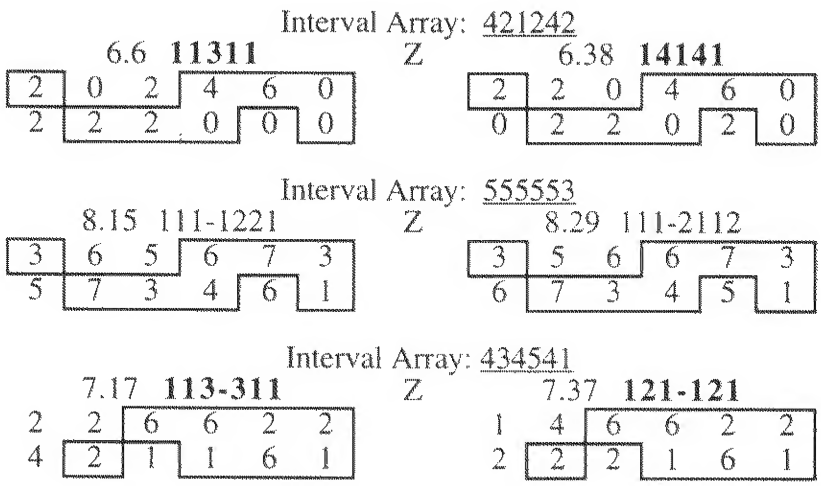


Fig. 2.6

And that, for the present, is all I have to say about Chromatic Map II.

There is a constant danger, of course, that any extended chart of abbreviated symbols will become so laborious to translate back into easily understandable meanings that it virtually defeats its own purpose. If I was occasionally in some doubt about the virtues of the preceding IPF chart (not for myself, but for others), I have been more so with regard to the present chart. People do seem to be using it, however, so I must take it that the (indeed relatively simple) shorthand involved here is, for some at least, quite possible to absorb without too much difficulty. Perhaps you will find it helpful, perhaps not. Looking at a map is one thing—exploring the actual territory is quite another.

CHART OF ARRAY STEERINGS

	4.1 I^4 **111**	4.2 112	4.3 IIm **121**	4.4 113	4.5 114	4.6 **115	4.7 $IIIm^4$ **131**	4.8 IVm^4 **141**	4.9 Vm^4 **151**	4.10 IIM^4 **212**	4.11 122	4.12 213	4.13 123	4.14 214	4.15 [Z29] 132	
1	**I**	**1**	**3**	**1**	**1**	**1**	**4**	**5**	**6**	[2]	[0]	[2]	[0]	[2]	[0]	1
2	**1**	**2**	[1]	[0]	[0]	[0]	--	--	--	**3**	**2** [1]	[0]	[1]	[0]	[4]	2
3	[0]	[1]	**1**	[2]	--	--	[1]	--	--	**2**	[0]	**3**	**3**	[0]	[1]	3
4	--	[0]	[0]	[1]	[2]	--	**1**	[1]	--	--	[1]	[2]	--	[3]	[0]	4
5	--	--	--	[0]	[1]	**5** [2]	[0]	**1**	**6** [1]	[0]	[0]	--	[1]	**5**[2]	[1]	5
6	--	--	--	--	[0]	[1]	--	[0]	**1**	--	--	[0]	[0]	--	[0]	6
I **1-1**	**1**	[0]	--	[0]	[0]	[0]	--	--	--	--	--	--	--	--	--	I **1-1**
IIm 1-2	[0]	[1]	[0]	--	--	--	--	--	--	[2]	[0]	--	[0]	--	--	IIm 1-2
IIM 2-1	[0]	--	[1]	--	--	--	--	--	--	[0]	--	[0]	--	[0]	--	IIM 2-1
IIIm 1-3	--	[0]	[0]	[1]	--	--	[0]	--	--	--	--	[2]	--	--	[0]	IIIm 1-3
IIIM 3-1	--	--	[0]	--	--	--	[1]	--	--	--	--	--	--	--	--	IIIM 3-1
IVm 1-4	--	--	--	[0]	[1]	--	[0]	[0]	--	--	[0]	--	--	[2]	--	IVm 1-4
IVM 4-1	--	--	--	--	--	--	[0]	[1]	--	--	--	--	--	--	--	IVM 4-1
Vm 1-5	--	--	--	--	[0]	[1]	--	[0]	**6**	--	--	--	[0]	--	[0]	Vm 1-5
VM 5-1	--	--	--	--	--	[7]	--	[0]	**6** [1]	--	--	--	--	--	--	VM 5-1
VI **2-2**	--	[0]	--	--	--	--	--	--	--	--	[1]	--	--	--	--	VI **2-2**
VIIm 2-3	--	--	--	[0]	--	--	--	--	--	[0]	--	--	[1]	--	--	VIIm 2-3
VIIM 3-2	--	--	--	--	--	--	--	--	--	[0]	[0]	--	--	--	[1]	VIIM 3-2
VIIIm2-4	--	--	--	--	[0]	--	--	--	--	--	--	[0]	--	--	--	VIIIm2-4
VIIIM4-2	--	--	--	--	--	--	--	--	--	--	--	--	--	--	[0]	VIIIM4-2
IX **5-5**	--	--	--	--	--	[2]	--	--	--	--	--	--	--	[2]	--	IX **5-5**
X **3-3**	--	--	--	--	--	--	--	--	--	--	--	[0]	[0]	--	--	X **3-3**
XIm 3-4	--	--	--	--	--	--	--	--	--	--	--	--	--	[0]	--	XIm 3-4
XIM 4-3	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	XIM 4-3
XII **4-4**	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	XII **4-4**

	4.16 142	4.17 IIIM **313**	4.18 133	4.19 144	4.20 IVM^4 **414**	4.21 VI^4 **222**	4.22 223	4.23 IX^4 VIIm^4 **232**	4.24 **224	4.25 VIIIm^4 **242**	4.26 VIIM^4 **323**	4.27 233	4.28 X^4	4.29 [Z15] 124	
1	[0]	[3]	[0]	[0]	[4]	--	--	--	--	--	--	--	--	[0]	**1**
2	[5]	--	--	--	--	VI	**2**	**5**	**2**	**6**	[3]	[0]	--	[1]	**2**
3	--	**4**	**3** [1]	[9]	[9]	--	[4]	[2]	--	--	**5**	**3** [2]	X^4	[0]	**3**
4	[1]	**3**	[0]	XII [1]	**5**	**2**	[0]	--	XII	**6** [2]	[8]	[8]	--	[3]	**4**
5	**5** [7]	[7]	[7]	[0]	**4**	--	**5** [2]	IX [2]	--	--	**3**	[0]	--	[7]	**5**
6	[1]	--	[1]	--	--	[0]	--	--	[2]	**2**	--	[2]	**3**	[1]	**6**
I **1-1**	--	--	--	--	--	--	--	--	--	--	--	--	--	--	I **1-1**
IIm 1-2	--	--	--	--	--	--	--	--	--	--	--	--	--	[0]	IIm 1-2
IIM 2-1	--	--	--	--	--	--	--	--	--	--	--	--	--	--	IIM 2-1
IIIm 1-3	--	[3]	[0]	--	--	--	--	--	--	--	--	--	--	--	IIIm 1-3
IIIM 3-1	--	[0]	--	[9]	--	--	--	--	--	--	--	--	--	--	IIIM 3-1
IVm 1-4	[0]	--	--	[0]	[4]	--	--	--	--	--	--	--	--	--	IVm 1-4
IVM 4-1	--	--	--	--	[0]	--	--	--	--	--	--	--	--	--	IVM 4-1
Vm 1-5	--	--	--	--	--	--	--	--	--	--	--	--	--	--	Vm 1-5
VM 5-1	[7]	--	[7]	--	--	--	--	--	--	--	--	--	--	[7]	VM 5-1
VI **2-2**	--	--	--	--	--	**2**	[0]	--	[0]	--	--	--	--	--	VI **2-2**
VIIm 2-3	--	--	--	--	--	--	[2]	[0]	--	--	[3]	[0]	--	--	VIIm 2-3
VIIM 3-2	--	--	--	--	--	--	--	[2]	--	--	[0]	--	--	--	VIIM 3-2
VIIIm2-4	--	--	--	--	--	[0]	--	--	[2]	**6**	--	--	--	[1]	VIIIm2-4
VIIIM4-2	[1]	--	--	--	--	[0]	--	--	[8]	**6** [2]	--	[8]	--	--	VIIIM4-2
IX **5-5**	[7]	--	--	--	--	--	[2]	**5** [2]	--	--	--	--	--	--	IX **5-5**
X **3-3**	--	--	[1]	--	--	--	--	--	--	--	--	[2]	X^4	--	X **3-3**
XIm 3-4	--	[0]	--	--	[9]	--	--	--	--	--	[5]	[5]	--	[0]	XIm 3-4
XIM 4-3	--	[0]	[0]	[5]	[5]	--	[0]	--	--	--	[8]	--	--	--	XIM 4-3
XII **4-4**	--	--	--	[1]	--	--	--	--	[0]	--	--	--	--	--	XII **4-4**

	5.1 **1111** I^5	5.2 1112	5.3 1121	5.4 1113	5.5 1114	5.6 1131	5.7 1141	5.8 **2112** SPIIM	5.9 1122	5.10 1212 II^5	5.11 2113	5.12 [Z36] **1221** SPIIm	5.13 1144	5.14 1132	5.15 **4114** SPIVM	5.16 1213	
1	I^4	I	IIIm	I	I	IVm	Vm	**1** [2]	**1**	**3**	**1**[2]	**5**	**1**	**1**	**1** [4]	**3**	1
2	I	IIm	**2**	**1**	**1**	[0]	[0]	VI	VI	**3** [1]	**2**	**2** [1]	**2** [10]	**5**	**6** [4]	[1]	2
3	**1**	**2**	**1** [1]	**3**	[0]	[2]	--	**3**	[1]	IIm	**4**	**3**	[10]	[2]	--	IIIm	3
4	[0]	[1]	**1**	[2]	[3]	**1** [1]	[2]	**2**	**2**	[0]	**3**	[1]	XII [2]	[1]	**6**	**3**	4
5	--	[0]	[0]	[1]	**5** [2]	**1**	Vm [1]	--	[1]	[1]	**5** [2]	**1**	[1]	IX [2]	**5**	[7]	5
6	--	--	--	[0]	[1]	[0]	**1**	[0]	[0]	[0]	--	[0]	[0]	[1]	**4**	[1]	6
I **1-1**	I	**1**	[0]	**1**	**1**	[0]	[0]	[2]	[0]	--	[2]	--	[0]	[0]	[4]	--	I **1-1**
IIm 1-2	**1**	**2**	[1]	[0]	[0]	--	--	[3]	[1]	**3**	--	[0]	--	--	--	[0]	IIm 1-2
IIM 2-1	**1**	[0]	[2]	[0]	[0]	--	--	[0]	--	[1]	[0]	[3]	[10]	--	--	[1]	IIM 2-1
IIIm 1-3	[0]	[1]	**1**	[2]	--	[1]	--	[2]	[0]	[0]	[3]	--	--	[1]	--	**3**	IIIm 1-3
IIIM 3-1	[0]	--	[1]	--	--	[2]	--	[0]	--	[0]	[0]	--	[10]	--	--	[0]	IIIM 3-1
IVm 1-4	--	[0]	[0]	[1]	[2]	**1**	[1]	--	[1]	--	[2]	[0]	[1]	[0]	[5]	--	IVm 1-4
IVM 4-1	--	--	[0]	--	--	[1]	[2]	--	--	--	--	[1]	--	--	[0]	--	IVM 4-1
Vm 1-5	--	--	--	[0]	[1]	[0]	Vm	--	[0]	[0]	--	[0]	[0]	[1]	[4]	--	Vm 1-5
VM 5-1	--	--	--	--	[7]	[0]	**6** [1]	--	--	--	--	[0]	--	[7]	[0]	[7]	VM 5-1
VI **2-2**	[0]	[1]	[0]	--	--	--	--	**2**	**2**	--	[0]	[1]	[10]	--	--	--	VI **2-2**
VIIm 2-3	--	[0]	[0]	[1]	--	[0]	--	--	--	[1]	[2]	[1]	--	[0]	--	--	VIIm 2-3
VIIM 3-2	--	[0]	--	--	--	--	--	--	[1]	[1]	--	[0]	--	[2]	--	--	VIIM 3-2
VIIIm2-4	--	--	--	[0]	[1]	[0]	[0]	[0]	[0]	--	--	--	[0]	--	**6** [4]	[1]	VIIIm 2-4
VIIIM4-2	--	--	--	--	--	--	--	[0]	[0]	[0]	--	--	[6]	[1]	**6**	--	VIIIM4-2
IX **5-5**	--	--	--	--	[2]	--	[2]	--	--	--	[2]	--	--	**5** [2]	[0]	--	IX **5-5**
X **3-3**	--	--	--	[0]	--	--	--	[0]	--	[0]	--	[0]	--	--	--	[1]	X **3-3**
XIm 3-4	--	--	--	--	[0]	--	--	--	--	--	[0]	--	--	--	--	[0]	XIm 3-4
XIM 4-3	--	--	--	--	--	--	--	--	--	--	[0]	--	[6]	--	--	[0]	XIM 4-3
XII **4-4**	--	--	--	--	--	--	--	--	--	--	--	--	[2]	--	--	--	XII **4-4**

	5.17 [Z37] **1441** SPIVm	5.18 [Z38] 1312	5.19 1231	5.20 1414	5.21 1313	5.22 **1331** SPIIIm	5.23 2122	5.24 1222	5.25 2123	5.26 2213	5.27 1223	5.28 2132	5.29 1232	5.30 1322	5.31 1333	
1	**3** [9]	**4**	**6**	**5**	**4**	**5** [7]	[2]	[0]	[2]	[4]	[0]	[2]	[0]	[0]	[0]	**1**
2	[10]	[5]	[1]	[10]	--	--	VIIM	VI [1]	**3**	**2**	**2** [1]	**6**	**5** [1]	**2** [4]	[10]	**2**
3	**1** [9]	**3** [1]	**3**	[10]	**4** [1]	**3** [1]	**2**	[0]	VIIm	**3** [2]	**5**	**3**	**3**	[1]	X^4 [1]	**3**
4	XII [1]	**1**	[3]	**5** [1]	441 [4]	XII	[3]	**2** [1]	[8]	XII	**5** [8]	**6** [2]	[8]	XII	[0]	**4**
5	**5**	**5** [7]	**6** [1]	IVm	**4** [8]	**1** [7]	IX [2]	**5** [7]	**3**	[0]	XIM [8]	[3]	IX [3]	**5** [8]	[7]	**5**
6	--	[1]	**1**	[0]	--	[1]	--	[1]	[2]	[2]	--	**2**	[0]	[0]	**3** [1]	**6**
I **1-1**	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	I **1-1**
IIm 1-2	[9]	[4]	[0]	--	--	--	[2]	[0]	[2]	--	[0]	--	[0]	--	--	IIm 1-2
IIM 2-1	[10]	--	--	[10]	--	--	[0]	--	[0]	[2]	--	[0]	--	--	[10]	IIM 2-1
IIIm 1-3	[9]	[0]	--	--	**4**	[0]	--	--	--	[4]	--	[2]	--	[0]	[0]	IIIm 1-3
IIIM 3-1	[9]	[1]	[3]	--	[1]	[4]	--	--	--	--	--	--	--	--	--	IIIM 3-1
IVm 1-4	[0]	[0]	--	**5**	[0]	[7]	[2]	[0]	--	--	[0]	--	--	--	--	IVm 1-4
IVM 4-1	[5]	[0]	--	[1]	**4** [8]	[8]	--	--	--	[0]	[8]	--	[8]	[8]	--	IVM 4-1
Vm 1-5	--	--	**6**	[0]	--	[7]	--	--	[2]	--	--	[2]	[0]	[0]	--	Vm 1-5
VM 5-1	--	[7]	**6** [1]	[0]	--	[7]	--	[7]	--	--	--	--	--	--	[7]	VM 5-1
VI **2-2**	--	--	--	--	--	--	[3]	**2** [1]	--	[0]	[1]	--	--	[4]	--	VI **2-2**
VIIm 2-3	--	--	[1]	--	--	--	[0]	--	**3**	[0]	[3]	--	[1]	--	--	VIIm 2-3
VIIM 3-2	--	--	--	--	--	--	**2**	[0]	[0]	--	[0]	[3]	[3]	[1]	[7]	VIIM 3-2
VIIIm2-4	--	--	[1]	--	--	--	--	[1]	--	[2]	--	**6**	[6]	[6]	[10]	VIIIm2-4
VIIIM4-2	--	[1]	--	[6]	--	--	--	[1]	[8]	[8]	--	**6** [2]	--	[0]	--	VIIIM4-2
IX **5-5**	[0]	[7]	--	[0]	--	--	**5** [2]	[7]	--	--	[3]	--	**5** [3]	[8]	--	IX **5-5**
X **3-3**	--	[1]	[0]	--	--	[1]	--	--	[2]	[2]	--	[0]	[0]	--	X^4 [1]	X **3-3**
XIm 3-4	[10]	--	[0]	[10]	**4** [1]	[1]	[0]	[0]	[5]	[5]	[5]	--	--	[1]	--	XIm 3-4
XIM 4-3	[5]	[0]	--	[6]	[1]	[0]	--	--	[8]	--	**5** [8]	[8]	[8]	--	[0]	XIM 4-3
XII **4-4**	[1]	--	--	--	[0]	[0]	--	--	--	[0]	--	--	--	[0]	--	XII **4-4**

	5.32 3133	5.33 2222 VI^5	5.34 2332 SPVIIm	5.35 IX^5 3223 SPVIIM	5.36 [Z12] 1123	5.37 [Z17] 3113 SPIIIM	5.38 [Z18] 1133	6.1 11111 I^6	6.2 11112	6.3 [Z36] 11121	6.4 {Z37] 11211	6.5 11131	6.6 [Z38] 11311	
1	[3]	--	--	--	1	1 [3]	1	I^5	I^4	113	IIIm^4	114	IVm^4	1
2	[10]	VI^4	VI [8]	IX [5]	2	[3]	[0]	I^4	112	IIm	VI	1	5	2
3	XIM	--	3 [2]	5 [7]	3 [1]	5	3 [2]	I	IIm	IIM	1 [1]	3	[2]	3
4	3	**224	2 [8]	[3]	[0]	XII	5 [8]	1	2	1 [1]	I	1 [2]	1 [1]	4
5	3 [7]	--	5	VIIm^4 [5]	5 [2]	3	4 [8]	[0]	[1]	1	1	Vm [1]	**115	5
6	[4]	2	[2]	--	[1]	--	[2]	--	[0]	[0]	[0]	1	1	6
I 1-1	--	--	--	--	[0]	[3]	[0]	I^4	I	1	4	1	5	I 1-1
IIm 1-2	--	--	--	--	[1]	--	--	I	IIm	2	[1]	[0]	--	IIm 1-2
IIM 2-1	--	--	--	--	--	--	--	I	1	3	[2]	[0]	--	IIM 2-1
IIIm 1-3	[3]	--	--	--	[0]	[4]	[1]	1	2	1 [1]	1	[2]	[1]	IIIm 1-3
IIIM 3-1	[0]	--	--	--	--	[0]	--	1	[0]	[2]	1 [1]	[3]	[2]	IIIM 3-1
IVm 1-4	--	--	--	--	--	[3]	[0]	[0]	[1]	1	1	1 [1]	1	IVm 1-4
IVM 4-1	--	--	--	--	--	[0]	[8]	[0]	--	[1]	1	[2]	1 [1]	IVM 4-1
Vm 1-5	--	--	--	--	[1]	--	--	--	[0]	[0]	[0]	Vm	Vm	Vm 1-5
VM 5-1	[10]	--	--	--	[7]	--	[8]	--	--	[0]	[0]	6 [1]	VM [7]	VM 5-1
VI 2-2	--	VI	2 [8]	[3]	[0]	--	--	1	2	[1]	2	--	--	VI 2-2
VIIm 2-3	[10]	--	[0]	5 [5]	[2]	[3]	[0]	[0]	[1]	1	[0]	[1]	[0]	VIIm 2-3
VIIM 3-2	[7]	--	[5]	5 [7]	--	[0]	--	[0]	[1]	[0]	[1]	--	[2]	VIIM 3-2
VIIIm2-4	[10]	VIIIm	[8]	--	--	--	--	--	[0]	[0]	[0]	1	[0]	VIIIm 2-4
VIIIM4-2	--	VIIIM [8]	[8]	--	--	--	[8]	--	[0]	--	[0]	--	[1]	VIIIM4-2
IX 5-5	--	--	[0]	IX [7]	[2]	--	--	--	--	--	--	[2]	5 [2]	IX 5-5
X 3-3	[4]	--	[2]	--	[1]	--	[2]	--	[0]	[0]	--	[0]	--	X 3-3
XIm 3-4	[0]	--	[5]	[0]	--	[5]	[5]	--	--	--	--	[0]	--	XIm 3-4
XIM 4-3	3	--	[10]	[3]	[0]	[8]	[1]	--	--	--	--	--	--	XIM 4-3
XII 4-4	--	[0]	--	--	--	[0]	--	--	--	--	--	--	--	XII 4-4

	6.7 Mode 5 **11411**	6.8 **21112**	6.9 11122	6.10 [Z39] 12112	6.11 [Z40] 11212	6.12 [Z41] 11221	6.13 [Z42] **12121** IIm^6	6.14 12113	6.15 11213	6.16 13112	6.17 [Z43] 11231	6.18 11321	6.19 [Z44] 12131	
1	Vm^4	I [2]	I	IIIM	IIIm	Vm	X	IIIM	IIIm	IVM	VM [7]	VM [7]	XIm	1
2	6	IIM^4	122	VI [1]	VIIm	VI	3 [1]	2 [1]	2	2 [4]	2	5	[1]	2
3	--	VI	2	IIIm	IIm [1]	3 [1]	IIm^4	IVm	IIIm [1]	4 [1]	3 [1]	3 [2]	IIIm	3
4	6 [2]	3	2 [1]	IIm	1	2	3	441 [4]	441 [4]	441 [4]	XII	5 [8]	144 [3]	4
5	Vm^4 [7]	IX [2]	IX [2]	5 [7]	IX [2]	Vm [1]	6 [1]	XIM [8]	4 [8]	IVM [8]	VM [2]	142 [7]	IVM [3]	5
6	I	--	[1]	[1]	[1]	1	1	--	[2]	[0]	1 [1]	1 [1]	[1]	6
I 1-1	6	1 [2]	1	[3]	[0]	[0]	--	[3]	[0]	[4]	[0]	[0]	--	I 1-1
IIm 1-2	--	2 [2]	2	4	3 [1]	[1]	3	[0]	[1]	[5]	[1]	--	[0]	IIm 1-2
IIM 2-1	--	2	[0]	[1]	[2]	[4]	3 [1]	[1]	[2]	--	--	[5]	[1]	IIM 2-1
IIIm 1-3	--	[3]	[1]	3	1	[0]	3	4	IIIm	4	[0]	[1]	3	IIIm 1-3
IIIM 3-1	--	[0]	--	1	[1]	--	3	1	[1]	[1]	[4]	--	4	IIIM 3-1
IVm 1-4	6 [1]	[2]	2	[0]	[0]	[1]	--	3	[0]	[0]	[7]	5 [7]	4 [3]	IVm 1-4
IVM 4-1	6 [2]	[0]	--	[0]	[0]	[2]	--	4 [8]	4 [8]	IVM [8]	[8]	[8]	5 [3]	IVM 4-1
Vm 1-5	Vm^4	--	[1]	--	[1]	Vm	6	--	--	[0]	6 [1]	6 [1]	[7]	Vm 1-5
VM 5-1	Vm^4 [7]	--	[7]	[7]	[7]	6 [1]	6 [1]	--	[8]	[0]	VM [2]	VM [2]	[7]	VM 5-1
VI 2-2	--	3	2 [1]	2 [1]	[0]	2	--	[1]	[0]	[4]	[0]	--	--	VI 2-2
VIIm 2-3	--	2	[0]	--	2	[2]	[1]	[3]	[0]	--	[2]	[0]	--	VIIm 2-3
VIIM 3-2	--	2	2	[0]	[2]	[1]	[1]	[0]	--	[1]	--	[2]	--	VIIM 3-2
VIIIm 2-4	6	--	[1]	[1]	--	[0]	[1]	--	[2]	[6]	[2]	--	[1]	VIIIm 2-4
VIIIM4-2	6 [2]	--	[1]	[1]	[1]	[0]	[0]	--	[8]	[0]	[8]	5 [8]	--	VIIIM4-2
IX 5-5	6 [2]	5 [2]	5 [2]	[7]	5 [2]	[2]	--	[3]	--	[8]	[2]	5 [2]	[3]	IX 5-5
X 3-3	--	--	--	[1]	[1]	[1]	1	--	[2]	--	[1]	[2]	[1]	X 3-3
XIm 3-4	--	[0]	[0]	[0]	--	--	[0]	4 [1]	4 [1]	4 [1]	[1]	[5]	1	XIm 3-4
XIM 4-3	--	[0]	--	[0]	[0]	[0]	[0]	5 [8]	[1]	[1]	[0]	[1]	4 [8]	XIM 4-3
XII 4-4	--	--	--	--	--	--	--	[0]	[0]	[0]	[0]	--	[0]	XII 4-4

	6.20 **13131** III^6	6.21 21122	6.22 11222	6.23 [Z45] **21212** IIM^6	6.24 [Z46] 12122	6.25 [Z47] 12212	6.26 [Z48] **12221**	6.27 12123	6.28 [Z49] **21331	6.29 [Z50] **12321**	6.30 12312	6.31 12231	
1	XII	**1** [2]	**1**	**3** [2]	**3**	**5**	**5** [7]	**3**	**5** [9]	**4** [8]	**6**	**4** [8]	1
2	--	VI^4	VI^4	X	VIIM [1]	VIIm [3]	VI [1]	**3** [1]	**2** [10]	**5** [1]	**6** [1]	**2** [1]	2
3	XII [1]	**3**	**3** [1]	IIM^4	IIm	VIIM	**5**	3331 [3]	X^4	X^4	X^4	XIM [5]	3
4	**13131**	**224	**224	**6** [2]	XII	**5** [8]	IX [3]	**3** [9]	XII [2]	**1** [8]	**6** [3]	144 [8]	4
5	XII	[3]	**5** [8]	**3**	IX [3]	412 [8]	IVM^4 [3]	**3** [1]	**1** [9]	IX [3]	**6** [1]	XIM [8]	5
6	--	**2**	**2**	**2**	[0]	[0]	[1]	**3**	**3**	**3**	IIm	[3]	6
I **1-1**	--	[2]	[0]	--	--	--	--	--	--	--	--	--	I **1-1**
IIm 1-2	--	[3]	[1]	**3** [2]	**3**	**5**	[0]	**3**	[9]	[0]	**6**	[0]	IIm 1-2
IIM 2-1	--	[0]	--	**3**	[1]	[3]	[5]	[1]	[0]	[6]	--	--	IIM 2-1
IIIm 1-3	XII	[2]	[0]	[2]	[0]	--	--	[0]	[2]	[8]	--	[8]	IIIm 1-3
IIIM 3-1	XII [1]	[0]	--	[2]	[0]	--	--	**3** [9]	[6]	[9]	**6** [3]	**4** [5]	IIIM 3-1
IVm 1-4	XII	[3]	[1]	--	[3]	[0]	**5** [7]	--	[9]	[8]	--	**4** [8]	IVm 1-4
IVM 4-1	XII	--	[8]	--	[8]	**5** [8]	**5** [3]	--	[10]	[8]	--	[8]	IVM 4-1
Vm 1-5	--	[2]	[0]	[2]	[0]	[0]	[7]	**3**	[9]	[0]	**6**	--	Vm 1-5
VM 5-1	--	--	[8]	[0]	--	[0]	[7]	--	[9]	[3]	**6** [1]	[3]	VM 5-1
VI **2-2**	--	VI	VI	--	[4]	[1]	**2** [1]	--	[10]	--	--	[1]	VI **2-2**
VIIm 2-3	--	--	--	**3**	[1]	**2** [1]	[3]	**3** [1]	[10]	[1]	**6** [1]	[3]	VIIm 2-3
VIIM 3-2	--	[3]	[1]	**3**	**2** [1]	**3**	[0]	[1]	[9]	[3]	--	[0]	VIIM 3-2
VIIIm 2-4	--	VIIIm	VIIIm	**6**	[6]	[6]	[1]	--	[0]	[6]	**6** [1]	[3]	VIIIm 2-4
VIIIM4-2	--	VIIIM [8]	VIIIM [8]	**6** [2]	[0]	--	[1]	**3** [9]	[6]	[9]	**6** [3]	[9]	VIIIM4-2
IX **5-5**	--	--	[8]	--	**5** [3]	**5** [3]	**4** [3]	--	--	**5** [3]	--	[3]	IX **5-5**
X **3-3**	--	[0]	--	**2**	[0]	[0]	--	X^4	X^4	X^4	X^4	[9]	X **3-3**
XIm 3-4	XII [1]	--	[1]	[5]	[1]	[5]	**5**	**3** [6]	[3]	[6]	**6**	[5]	XIm 3-4
XIM 4-3	XII [1]	[8]	--	[8]	[8]	**5** [8]	**5** [8]	[9]	[2]	[8]	--	XIM [1]	XIM 4-3
XII **4-4**	**2**	[0]	[0]	--	[0]	--	--	--	[2]	--	--	[1]	XII **4-4**

	6.32 IX^6 22122	6.33 21222	6.34 12222	6.35 Mode 1 22222 VI^6	6.36 [Z3] 11113	6.37 [Z4] **11441	6.38 [Z6] 14141 IVm^6	6.39 [Z10] 21113	6.40 [Z11] 11123	6.41 [Z12] 11132	6.42 [Z13] 31113	6.43 [Z17] 11312	
1	[4]	[2]	[0]	--	I^4 [1]	I^4 [10]	**115 [10]	I [2]	I	I	I [3]	IVm	1
2	VIIm^4	322	VI^4 [1]	VI^6	I	I [10]	1 [10]	IIM	IIm	Vm	1 [3]	6	2
3	IX [4]	VIIM [9]	3 [9]	--	IIIm	1 [10]	[10]	VIIm	VIIm	3	X^4	3 [2]	3
4	5	2 [3]	**224	VI^6	3	XII [2]	5 [1]	XII	5 [8]	6 [2]	5	VM [8]	4
5	SPVIIM [9]	VIIm^4 [7]	5 [7]	--	5 [2]	5 [1]	IVm^4	3	XIM [8]	IX [3]	4	IVM [8]	5
6	--	[3]	2 [1]	VI	[1]	[0]	5	[2]	[2]	2	3	2	6
I 1-1	--	--	--	--	I	I [10]	1 [10]	1 [2]	1	1	1 [3]	[0]	I 1-1
IIm 1-2	[4]	[2]	[0]	--	1	1 [10]	[10]	[2]	2	[0]	[3]	[5]	IIm 1-2
IIM 2-1	[2]	[0]	--	--	1	1 [10]	[10]	2	[0]	[0]	[3]	--	IIM 2-1
IIIm 1-3	--	--	--	--	3	[10]	--	[4]	[1]	[2]	[5]	[1]	IIIm 1-3
IIIM 3-1	--	--	[9]	--	[0]	[10]	--	[0]	--	--	[0]	[2]	IIIM 3-1
IVm 1-4	[4]	[2]	[0]	--	[2]	[1]	5	[3]	[0]	[1]	[4]	1	IVm 1-4
IVM 4-1	[0]	--	--	--	--	[6]	5 [1]	[0]	[8]	[8]	[0]	5 [8]	IVM 4-1
Vm 1-5	--	--	--	--	[2]	[0]	Vm [11]	[2]	[2]	2	[3]	[0]	Vm 1-5
VM 5-1	--	[9]	[7]	--	[7]	[6]	VM	--	[8]	[8]	[0]	4 [8]	VM 5-1
VI 2-2	5	2 [3]	VI [1]	VI^6	[0]	[10]	--	[0]	[1]	--	--	--	VI 2-2
VIIm 2-3	IX [2]	5 [7]	[7]	--	[2]	--	--	3	3	[1]	[4]	[0]	VIIm 2-3
VIIM 3-2	IX [4]	VIIM [9]	[0]	--	--	--	--	[0]	[0]	[3]	[0]	--	VIIM 3-2
VIIIm 2-4	--	[3]	VIIIm [1]	VI^6	[1]	[0]	[11]	[2]	--	6	[3]	6	VIIIm 2-4
VIIIM4-2	--	[3]	VIIIM [9]	VI^6	--	[6]	[6]	[8]	[8]	6 [2]	[0]	6 [2]	VIIIM4-2
IX 5-5	VIIm^4 [2]	IX [9]	[7]	--	[2]	[1]	1	--	[3]	5 [3]	--	[8]	IX 5-5
X 3-3	--	[9]	[9]	--	[1]	--	--	[2]	[2]	[0]	X^4	[2]	X 3-3
XIm 3-4	5 [9]	2	[0]	--	[0]	[11]	[10]	[5]	[5]	--	[5]	[5]	XIm 3-4
XIM 4-3	5	[5]	[5]	--	[0]	[6]	[6]	[8]	5 [8]	[8]	[9]	[1]	XIM 4-3
XII 4-4	--	--	[1]	2	--	[2]	--	[0]	--	--	--	--	XII 4-4

	6.44 [Z19] 11313	6.45 [Z23] **12332	6.46 [Z24] 11223	6.47 [Z25] 11232	6.48 [Z26] **13223	6.49 [Z28] **13231**	6.50 [Z29] **23132**	7.1 **111-111** I^7	7.2 111-112	7.3 111-113	7.4 111-121	
1	IVm	**1** [11]	**1**	**1**	**1** [11]	**3** [9]	**6** [5]	I^6	I^5	I^5	1113	1
2	[0]	VI [9]	VI	IX [2]	IX [6]	**6** [4]	**3** [9]	I^5	1112	I^4	112	2
3	XIM	X^4	XIm [6]	XIM [9]	**5** [8]	$IIIM^4$ [6]	$VIIM^4$ [6]	I^4	112	113	IIm^4	3
4	441 [5]	**2** [9]	VIIM [9]	**3** [9]	XII	X [6]	**3** [2]	I	IIm	441 [4]	IIM	4
5	IIIM [9]	**5** [1]	XIM [9]	$VIIm^4$ [2]	$VIIm^4$ [6]	**3** [1]	X [6]	**1**	IX [2]	XIM [8]	Vm [1]	5
6	[0]	**3**	[0]	[1]	[0]	**4**	**5**	[0]	[1]	[2]	**1**	6
I 1-1	[0]	[11]	[0]	[0]	[11]	--	--	I^5	I^4	I^4	I	I 1-1
IIm 1-2	--	[0]	[1]	[1]	--	[9]	[11]	I^4	112	I	IIm	IIm 1-2
IIM 2-1	--	[9]	--	--	--	[10]	[9]	I^4	I	I	IIIm	IIM 2-1
IIIm 1-3	**4** [1]	[11]	[0]	[0]	[0]	**3** [9]	[5]	I	IIm	IIIm	IIM	IIIm 1-3
IIIM 3-1	**5** [9]	[9]	[9]	[9]	[8]	**3** [6]	[2]	I	**1**	**1**	**3**	IIIM 3-1
IVm 1-4	**1**	--	[1]	--	[11]	--	--	**1**	**2**	**3**	**1** [1]	IVm 1-4
IVM 4-1	**4** [9]	--	[9]	[9]	[8]	--	--	**1**	[0]	**4** [8]	[2]	IVM 4-1
Vm 1-5	[0]	[0]	[0]	[1]	[0]	[0]	**6** [5]	[0]	[1]	[2]	Vm	Vm 1-5
VM 5-1	[0]	[6]	--	[7]	[6]	[4]	**6**	[0]	[7]	[8]	**6** [1]	VM 5-1
VI 2-2	--	**2** [9]	**2**	[0]	[4]	--	--	I	IIm	**1**	**2**	VI 2-2
VIIm 2-3	[0]	[1]	[4]	**5** [2]	**5** [6]	[4]	**3** [9]	**1**	**2**	**3**	**1** [1]	VIIm 2-3
VIIM 3-2	[9]	[6]	**4** [9]	**5** [4]	**5** [8]	[1]	**3** [6]	**1**	**2**	[0]	[1]	VIIM 3-2
VIIIm 2-4	[0]	[9]	[0]	[7]	[6]	**6** [4]	[0]	[0]	[1]	[2]	**1**	VIIIm 2-4
VIIIM4-2	--	[9]	[0]	--	[0]	**6**	[5]	[0]	[1]	[8]	[0]	VIIIM 4-2
IX 5-5	--	[1]	[4]	IX [4]	IX [8]	--	--	--	**5** [2]	[3]	[2]	IX 5-5
X 3-3	[6]	X^4	[6]	[1]	--	**4** [6]	**5** [6]	[0]	[1]	[2]	**1**	X 3-3
XIm 3-4	**4** [2]	[6]	**3** [6]	[9]	[1]	**3** [6]	**3** [11]	--	[0]	**4** [1]	[0]	XIm 3-4
XIM 4-3	**3** [2]	[11]	**5** [9]	**3** [9]	[4]	**3** [6]	**3** [2]	--	[0]	**5** [8]	[0]	XIM 4-3
XII 4-4	[1]	--	--	--	[0]	--	--	--	--	[0]	--	XII 4-4

	7.5 111-211	7.6 111-131	7.7 111-311	7.8 211-112	7.9 111-122	7.10 111-133	7.11 121-112	7.12 [Z36] 311-113	7.13 112-112	7.14 111-221	7.15 112-211	
1	1131	1114	1141	I^{4} [2]	I^{4}	I^{4}	311	I^{4} [3]	IIIm^{4}	**115	Vm^{4}	1
2	122	I	Vm	SPIIM	1122	211 [10]	IIM^{4} [1]	**115 [3]	VI^{4}	122	VI^{4}	2
3	IIM	IIIm	3	IIM^{4}	IIM	1333	122	IIIM^{4}	IIIM [1]	VIIm	3 [1]	3
4	I [1]	144 [3]	Vm [2]	**224	**224	VIIm [10]	441 [4]	X	4411 [4]	IX [3]	**224	4
5	**115	IVm^{4} [2]	1141 [1]	3	IX [3]	XIm [7]	412 [8]	VIIm^{4} [5]	IVM [8]	1414 [2]	Vm^{4} [7]	5
6	1	1 [1]	I	2	2	3 [1]	[0]	4	2	1 [1]	I	6
I 1-1	IVm	I	Vm	I [2]	I	I	1 [3]	I [3]	4	1	6	I 1-1
IIm 1-2	2	1	[0]	IIm [2]	IIm	1	VIIM	1 [3]	4 [1]	2	[1]	IIm 1-2
IIM 2-1	3	1	[0]	IIM	1	IIM [10]	2 [1]	1 [3]	[2]	5	[4]	IIM 2-1
IIIm 1-3	1 [1]	3	[2]	2 [2]	2	3	4	3 [3]	IIIm	[1]	[0]	IIIm 1-3
IIIM 3-1	1 [2]	4	[3]	2	[0]	2 [10]	1	3	1 [1]	--	[4]	IIIM 3-1
IVm 1-4	I	IVm [2]	Vm [1]	[3]	2 [1]	[2]	3	[5]	1	IX [2]	6 [1]	IVm 1-4
IVM 4-1	1 [1]	5 [3]	VM [2]	[0]	[8]	[10]	IVM [8]	[0]	IVM [8]	5 [3]	6 [2]	IVM 4-1
Vm 1-5	Vm	Vm [1]	1141	[2]	2	[1]	[0]	2 [4]	[0]	Vm [1]	Vm^{4}	Vm 1-5
VM 5-1	VM [7]	VM [2]	Vm^{4} [1]	[0]	[8]	3 [7]	[0]	2 [10]	4 [8]	VM [2]	Vm^{4} [7]	VM 5-1
VI 2-2	2 [1]	[0]	--	VI	VI	2 [10]	3 [1]	[3]	VI	2 [1]	VI	VI 2-2
VIIm 2-3	1	[2]	[1]	3	[1]	4 [10]	2 [1]	5 [5]	[0]	3	[2]	VIIm 2-3
VIIM 3-2	2	--	[3]	3	2 [1]	3 [7]	IIm	5 [7]	[1]	2	[1]	VIIM 3-2
VIIIm 2-4	1	1 [1]	Vm	VIIIm	VIIIm	3 [10]	[6]	6 [4]	VIIIm	[1]	VIIIm	VIIIm 2-4
VIIIM4-2	[1]	[8]	6 [2]	VIIIM [8]	VIIIM [8]	[10]	[0]	6	VIIIM [8]	5 [8]	VIIIM [8]	VIIIM4-2
IX 5-5	5 [2]	1 [2]	Vm [2]	--	5 [3]	[2]	5 [3]	IX [7]	[8]	IVm [2]	6 [2]	IX 5-5
X 3-3	[0]	[1]	[0]	2	[0]	X^{4} [1]	[0]	4	[2]	[2]	[1]	X 3-3
XIm 3-4	[0]	1	[0]	[5]	[1]	5 [7]	4 [1]	3	4 [1]	5	[1]	XIm 3-4
XIM 4-3	--	4 [8]	[8]	[8]	[8]	3	5 [8]	3	[1]	5 [8]	[0]	XIM 4-3
XII 4-4	--	[0]	--	[0]	[0]	--	[0]	--	[0]	--	[0]	XII 4-4

	7.16 111-331	7.17 [Z37] **113-311**	7.18 [Z38] 111-313	7.19 131-113	7.20 113-113	7.21 113-131	7.22 **131-131**	7.23 211-122	7.24 111-222	7.25 211-212	
1	311 [9]	IIIm^4 [8]	114	411	IVm^4	144	IVM^4	I [2]	I	IIIm [2]	1
2	IIM [10]	VI [8]	IIM [10]	Vm [4]	IX	**2** [10]	**6** [4]	2122	1222	223	2
3	3331	IVM^4 [5]	VIIM^4[7]	X^4 [1]	XIm [7]	144 [9]	IIIM^4 [6]	322 [9]	VIIM [9]	3331 [6]	3
4	144 [9]	SPIIIM [5]	441 [6]	Vm	144 [1]	13131 [1]	SPIIIm [5]	VIIM	**224 [1]	IIM	4
5	IIIM [9]	IIIM^4 [5]	331 [7]	142	1132	144	IIIm^4	SPVIIM [9]	VIIm^4 [7]	VIIm^4 [2]	5
6	**3**	[2]	**1**	IIIm	**1**	[0]	**4**	[3]	**2** [1]	**3**	6
I 1-1	**1**	**4** [8]	**1**	**1** [4]	**5**	[0]	[4]	**1** [2]	**1**	[2]	I 1-1
IIm 1-2	**3** [9]	[9]	[0]	[4]	--	[9]	[9]	**2** [2]	**2**	**3** [3]	IIm 1-2
IIM 2-1	**2** [10]	[10]	**2** [10]	**6** [4]	[10]	[10]	[10]	**2**	[0]	**4**	IIM 2-1
IIIm 1-3	**5** [9]	IVm [6]	**4** [2]	**6**	**5** [1]	XII [1]	XIM [5]	[3]	[1]	**1** [2]	IIIm 1-3
IIIM 3-1	IIIM [6]	IVM [5]	**5** [10]	[1]	**4** [10]	441 [2]	XIm [6]	[0]	[9]	**3**	IIIM 3-1
IVm 1-4	**4** [9]	IIIm [8]	**1** [1]	**5**	IVm	144	IVM	**2** [2]	**2**	[2]	IVm 1-4
IVM 4-1	**1** [9]	IIIM [5]	**4** [10]	**1**	**1** [1]	XII [1]	IVm	[0]	[9]	[2]	IVM 4-1
Vm 1-5	**3** [9]	[8]	Vm	VIIIM	Vm	[0]	**4**	[3]	[1]	**3** [3]	Vm 1-5
VM 5-1	[9]	[8]	**6** [1]	VM [7]	VM [7]	[0]	**4**	[9]	**2** [7]	[9]	VM 5-1
VI 2-2	[10]	**2** [8]	[10]	--	[10]	[10]	--	VIIM	VI [1]	**2**	VI 2-2
VIIm 2-3	**3** [10]	**4** [8]	**3** [10]	[5]	**5**	[0]	[4]	IX [2]	**5** [7]	VIIm [2]	VIIm 2-3
VIIM 3-2	**1** [9]	**4** [5]	**3** [7]	**6** [1]	**5** [2]	[9]	[1]	322 [9]	VIIM [9]	**5** [4]	VIIM 3-2
VIIIm 2-4	[0]	[8]	**1**	**6** [4]	[0]	[0]	**6** [4]	[3]	VIIIm [1]	[0]	VIIIm 2-4
VIIIM 4-2	**3** [6]	[8]	[6]	Vm	**5** [1]	[6]	**6**	[3]	VIIIM [9]	**3**	VIIIM 4-2
IX 5-5	--	[0]	[2]	**5** [7]	IX [2]	[0]	[0]	VIIm^4 [2]	IX [9]	IX [4]	IX 5-5
X 3-3	X^4	[2]	**5** [7]	X^4 [1]	[7]	[6]	**4** [6]	[9]	[9]	X^4	X 3-3
XIm 3-4	**3** [3]	XIM [10]	XIm	[10]	**3** [7]	XII [2]	IIIM [6]	VIIM [9]	**2**	**3** [9]	XIm 3-4
XIM 4-3	**4** [2]	XIm [10]	**3** [3]	**6**	**4** [6]	144 [5]	IIIm [5]	**5**	[5]	**2**	XIM 4-3
XII 4-4	[2]	[1]	[2]	--	[2]	**1** [1]	[1]	--	[1]	--	XII 4-4

	7.26 121-122	7.27 112-122	7.28 122-112	7.29 112-212	7.30 112-221	7.31 121212	7.32 121-221	7.33 **221-122**	7.34 **122-221**	
1	IIIM	IIIm	VM	Vm	IVM [8]	X	XIM [8]	**1** [4]	**3** [9]	1
2	VI^4 [1]	VIIm^4	VI^4 [1]	223	VI^4	X [1]	VIIM [1]	VI^6	SPVIIm [5]	2
3	IIIM^4 [9]	412 [9]	X^4	VIIM^4 [1]	XIm [6]	12123	3331 [3]	**3** [2]	IIM^4 [7]	3
4	3122 [9]	144	**224 [1]	VIIM [9]	1322 [8]	X [9]	441	VI^6	**224 [1]	4
5	XIM	SPVIIM [9]	VM [7]	1232 [1]	IVM [4]	X [1]	214 [1]	**5**	VIIm^4 [5]	5
6	**2** [1]	[1]	IIm	**1**	**2**	IIm	**3**	VI	**2** [1]	6
I 1-1	[3]	[0]	[5]	[0]	[0]	--	--	[4]	--	I 1-1
IIm 1-2	**4**	**3** [1]	**6**	**5** [1]	[1]	X	**3**	[5]	**3** [9]	IIm 1-2
IIM 2-1	[1]	[2]	[3]	[4]	[6]	**3** [1]	**5** [1]	[2]	**3** [7]	IIM 2-1
IIIm 1-3	**3**	**1**	[5]	[0]	**4** [8]	**3**	**4** [8]	[4]	[9]	IIIm 1-3
IIIM 3-1	IIIM [9]	**4** [9]	**6** [3]	[9]	[9]	X [9]	**3** [9]	[2]	[9]	IIIM 3-1
IVm 1-4	**4**	**4**	[0]	[1]	**5** [8]	--	**5** [3]	[5]	[0]	IVm 1-4
IVM 4-1	[0]	**3** [9]	[1]	**5** [9]	IVM [4]	--	**4** [4]	[0]	[5]	IVM 4-1
Vm 1-5	[3]	[1]	**6**	Vm	**4** [8]	X	**3**	[4]	[9]	Vm 1-5
VM 5-1	[7]	[7]	VM [7]	**6** [1]	[8]	**6** [1]	[3]	[0]	[7]	VM 5-1
VI 2-2	VI [1]	**5**	VI [1]	**2**	VI	--	[4]	VI^6	VI [1]	VI 2-2
VIIm 2-3	[7]	IX [2]	**6** [1]	VIIm [2]	[4]	X [1]	**3** [1]	[0]	VIIm [5]	VIIm 2-3
VIIM 3-2	**4**	IX [4]	[0]	XIM [9]	**4** [9]	**3** [1]	**2** [1]	[5]	VIIM [7]	VIIM 3-2
VIIIm 2-4	VIIIm [1]	[7]	VIIIm [1]	**5** [7]	VIIIm	**6** [1]	[6]	VI^6	VIIIm [1]	VIIIm 2-4
VIIIM 4-2	VIIIM [9]	[1]	VIIIM [9]	[0]	VIIIM [8]	X [9]	**3** [9]	VI^6	VIIIM [9]	VIIIM 4-2
IX 5-5	[7]	VIIm^4 [2]	[7]	IX [4]	**4** [4]	--	**5** [3]	[0]	IX [7]	IX 5-5
X 3-3	**4** [9]	[1]	X^4	**5** [1]	[6]	3331 [3]	X^4	[2]	**2** [7]	X 3-3
XIm 3-4	**3** [9]	**5** [9]	**6**	**3** [6]	XIm [6]	X [6]	XIm [6]	[5]	**2** [10]	XIm 3-4
XIM 4-3	XIM [5]	XIM [5]	[5]	VIIM [9]	**5** [9]	**3** [9]	**1** [8]	[10]	**2** [3]	XIM 4-3
XII 4-4	[1]	[1]	[1]	--	[0]	--	[0]	**2**	[1]	XII 4-4

	7.35 IX^7 **212-212**	7.36 [Z12] 111-212	7.37 [Z17] **121-121**	7.38 [Z18] 112-121	8.1 I^8 **111-1-111**	8.2 1111-112	8.3 **111-1113	8.4 1111-121	8.5 1111-211	
1	**5** [2]	113	IIIM^4	133	I^7	I^6	I^6	11113	11131	1
2	SPVIIM [5]	123	VI [1]	VIIm	I^6	11112	I^5	1112	1122	2
3	VIIm^4	IIM^4	IIIm^4	IIm^4 [1]	I^5	1112	**31113** [9]	1121	IIm^4	3
4	IX [10]	VM [8]	SPIVm [3]	441 [4]	I^4	4411 [4]	**3113** [9]	SPIVm [3]	1144 [2]	4
5	**22122** [10]	412 [8]	IVM^4 [3]	142 [7]	**115	412 [8]	IIIM^4 [9]	1414 [2]	1141 [1]	5
6	[2]	**2**	[1]	**1** [1]	**1**	**2**	**3**	**1**	I	6
I 1-1	--	**1**	[3]	[0]	I^6	I^5	I^5	I^4	114	I 1-1
IIm 1-2	**5** [2]	VIIm	**4**	**3** [1]	I^5	1112	I^4	112	IIm	IIm 1-2
IIM 2-1	**5**	**3**	**4** [1]	**3** [2]	I^5	I^4	I^4	113	IIIm	IIM 2-1
IIIm 1-3	--	**1** [1]	IIIM	IIIm	I^4	112	113	IIm^4	IIM	IIIm 1-3
IIIM 3-1	--	[2]	IIIm	**3** [1]	I^4	I	311 [9]	IIIm	IIIM	IIIM 3-1
IVm 1-4	**5** [2]	**1**	XIm	**5** [7]	I	IIm	IIIm	214	114 [1]	IVm 1-4
IVM 4-1	**5** [10]	**5** [8]	XIM [8]	**4** [8]	I	IVM [8]	IIIM [9]	XIM [8]	Vm [2]	IVM 4-1
Vm 1-5	[2]	**2**	[7]	**6** [1]	Vm	**2**	**3**	Vm [1]	1141	Vm 1-5
VM 5-1	[2]	**4** [8]	[7]	VM [2]	VM [7]	**4** [8]	**3** [9]	VM [2]	Vm^4 [7]	VM 5-1
VI 2-2	IX [10]	[1]	**2** [1]	[0]	I^4	112	I	IIm	VI	VI 2-2
VIIm 2-3	VIIm^4 [10]	IIm	[3]	**2**	I	IIm	IIIm	IIM	**1** [1]	VIIm 2-3
VIIM 3-2	VIIm^4	**3**	[0]	[2]	I	IIm	IIIM [9]	**2**	**2** [1]	VIIM 3-2
VIIIm 2-4	[8]	**6**	[1]	[2]	**1**	VIIIm	**3**	**1** [1]	114	VIIIm 2-4
VIIIM 4-2	[8]	**6** [2]	[1]	**5** [8]	**1**	VIIIM [8]	**3** [9]	**5** [8]	VIIIM [8]	VIIIM 4-2
IX 5-5	SPVIIM [7]	**5** [3]	**4** [3]	**5** [2]	**5** [2]	**5** [3]	[4]	IVm [2]	Vm [2]	IX 5-5
X 3-3	[2]	**2**	[1]	**1** [1]	**1**	**2**	X^4	**1** [1]	**1**	X 3-3
XIm 3-4	IX [7]	[7]	IVm	**4** [1]	[0]	**4** [1]	XIM [2]	IVm	**1**	XIm 3-4
XIM 4-3	IX [10]	**5** [8]	IVM [8]	**1**	[0]	**5** [8]	XIm [2]	IVM [8]	**4** [8]	XIM 4-3
XII 4-4	--	--	[0]	[0]	--	[0]	[1]	[0]	[0]	XII 4-4

	8.6 111-2-111	8.7 3-11111-3	8.8 **1131-131	8.9 Mode 4 111-3-111	8.10 2-11111-2	8.11 1111-122	8.12 121-1112	8.13 1111-212	8.14 112-1112	
1	11311	**11114 [3]	14141	11411	I^5 [2]	I^5	3111	1113	1311	1
2	SPIIm	I^4 [3]	**115 [10]	Vm^4	21112	11122	SPIIM [1]	1123	2212	2
3	IIM^4	SPIIIM	IIIM^4 [7]	X^4	**21123	3112 [9]	12123	12123	1223 [1]	3
4	**115 [1]	13131 [11]	SPIIIm [6]	Vm^4	IIM^4	3122 [9]	3122 [9]	321 [9]	SPIIIM [9]	4
5	31113 [8]	SPIIIm [11]	11311	11411 [1]	SPVIIM [9]	SPVIIM [9]	133	1232 [1]	11232	5
6	I	5	IVm	I^4	3	2 [1]	IIm	IIm	1	6
I 1-1	IVm^4	I^4 [3]	**115 [10]	Vm^4	I^4 [2]	I^4	I [3]	I	IVM	I 1-1
IIm 1-2	VIIm	I [3]	1 [10]	6	112 [2]	112	312	123	VIIM [1]	IIm 1-2
IIM 2-1	VIIM	I [3]	1 [10]	6	211	I	IIM [1]	IIIm	2 [2]	IIM 2-1
IIIm 1-3	1 [1]	441 [7]	XIM [6]	6 [2]	IIm [2]	IIm	VIIM	IIM	IVm	IIIm 1-3
IIIM 3-1	1 [2]	144 [3]	XIm [7]	6 [3]	IIM	IIIM [9]	312 [9]	X [9]	IVM [9]	IIIM 3-1
IVm 1-4	**115	144 [6]	IVm^4	Vm^4 [1]	2 [2]	VI	4	1 [1]	IIIm	IVm 1-4
IVM 4-1	**115 [1]	441 [3]	IVM^4 [1]	Vm^4 [2]	2	3 [9]	2	5 [9]	311 [9]	IVM 4-1
Vm 1-5	1141	Vm [5]	1141 [11]	11411	3 [3]	2 [1]	X	123	Vm	Vm 1-5
VM 5-1	1411 [7]	VM [6]	1411	11411 [1]	3 [9]	2 [7]	VM [7]	VIIIm [7]	VM [7]	VM 5-1
VI 2-2	2 [1]	1 [3]	[10]	--	IIM^4	122	VI [1]	2	VIIm	VI 2-2
VIIm 2-3	IIm	3 [3]	5	6 [1]	223	IX [2]	X [1]	123 [1]	223	VIIm 2-3
VIIM 3-2	IIM	3	5 [2]	6 [3]	322 [9]	322	IIIm	XIM [9]	412 [9]	VIIM 3-2
VIIIm 2-4	Vm	1 [5]	Vm [11]	Vm^4	3	VIIIm [1]	VIIIm [1]	VM [7]	5 [7]	VIIIm 2-4
VIIIM 4-2	VM [8]	1 [11]	VM [1]	Vm^4 [2]	3	VIIIM [9]	312 [9]	X [9]	1	VIIIM 4-2
IX 5-5	IVm^4 [2]	1 [6]	**115	Vm^4 [2]	VIIm^4 [2]	VIIm^4 [2]	[7]	IX [4]	VIIm^4 [2]	IX 5-5
X 3-3	2	5	4 [7]	X^4	X^4	4 [9]	3331 [3]	3331 [3]	5 [1]	X 3-3
XIm 3-4	5	441 [8]	IIIM [7]	6	VIIm [9]	VIIM [9]	X [6]	X [6]	XIM [2]	XIm 3-4
XIM 4-3	5 [8]	144 [11]	IIIm [6]	6 [2]	VIIm	XIM [5]	XIM [5]	VIIM [9]	VIIM^4 [9]	XIM 4-3
XII 4-4	--	3	[2]	--	--	[1]	[1]	--	[1]	XII 4-4

	8.15 [Z29] 111-1221	8.16 111-2211	8.17 **121-1-121**	8.18 111-2121	8.19 1121-121	8.20 **112-1-211**	8.21 **1122-221	8.22 111-2122	
1	4111 [8]	1411 [7]	SPIIIM	1133	1313	SPIIIm	I^4 [10]	113	**1**
2	1122	1222	IIM^4 [1]	123	VI^4	VIIm^4	**221-122** [6]	22212 [6]	2
3	3331 [3]	VIIM^4 [9]	**12213	32121 [9]	1313 [1]	SPIVm [4]	IIM^4 [8]	2212 [10]	3
4	1322 [8]	2231 [1]	**13131**	SPIIIm [1]	1313-11 [4]	**13131**	VI^6	2231 [6]	4
5	1414 [3]	11321 [7]	SPIVm [3]	1312 [8]	1313 [8]	**31132 [4]	VIIm^4 [6]	**22122** [8]	5
6	IIM	I [1]	**3**	IIM	**2**	**1** [1]	VI	**2**	6
I **1-1**	I	VM [7]	**1** [3]	**1**	**4**	**5** [7]	I [10]	**1**	I **1-1**
IIm 1-2	IIm	**2**	VIIM	VIIm	**4** [1]	**3** [1]	IIm [10]	VIIm	IIm 1-2
IIM 2-1	Vm	**5**	VIIm [1]	X	**4** [2]	**3** [2]	IIM [8]	VIIm	IIM 2-1
IIIm 1-3	VIIIM [8]	**5** [8]	441 [8]	133 [1]	1313	441 [4]	**2** [10]	**1** [1]	IIIm 1-3
IIIM 3-1	**3** [9]	**4** [5]	144	XIM [1]	441 [5]	144 [4]	**2** [8]	**4** [10]	IIIM 3-1
IVm 1-4	IX [3]	142 [7]	144 [3]	IVM [8]	441 [4]	144 [7]	**2** [11]	IVm	IVm 1-4
IVM 4-1	IVM [4]	VM [3]	441 [4]	IVm [8]	1313 [8]	441	**2** [6]	VIIm [8]	IVM 4-1
Vm 1-5	421 [8]	Vm^4 [1]	**3**	VIIIM [8]	**4** [8]	Vm [7]	**2** [10]	**2**	Vm 1-5
VM 5-1	VM [3]	1411 [2]	**3**	133 [8]	**4** [8]	VM [2]	**2** [6]	4 [8]	VM 5-1
VI **2-2**	VI	VI [1]	**3** [1]	[1]	VI	**5**	VI^6	223 [6]	VI **2-2**
VIIm 2-3	**3** [1]	XIm	IIM [1]	IIm	**4**	IX [2]	VIIm [6]	2212 [8]	VIIm 2-3
VIIM 3-2	VIIIM [9]	VIIM [9]	IIm	X [9]	**4** [9]	IX [4]	VIIM [8]	VIIm^4 [10]	VIIM 3-2
VIIIm 2-4	VIIIm	VIIIm [1]	**3** [3]	X	VIIIm	**5** [2]	VI^6	VIIIm [6]	VIIIm 2-4
VIIIM 4-2	132 [8]	142 [8]	**3** [9]	Vm [8]	VIIIM [8]	**5** [8]	VI^6	VIIIM [2]	VIIIM 4-2
IX **5-5**	IVm [3]	142 [2]	**5** [3]	**5** [3]	**4** [4]	VIIm^4 [2]	IX [8]	SPVIIM [5]	IX **5-5**
X **3-3**	X^4	**5** [9]	X^4	1333 [2]	**4** [2]	**1** [1]	**2** [8]	**2**	X **3-3**
XIm 3-4	XIm [6]	VIIm	441 [9]	IIIM [2]	1313 [1]	441 [5]	**2** [11]	IX [5]	XIm 3-4
XIM 4-3	Vm [8]	XIM [1]	144 [8]	133 [1]	441 [5]	144	**2** [4]	223 [6]	XIM 4-3
XII **4-4**	[0]	[1]	**1**	[1]	**1**	**1**	**2**	[2]	XII **4-4**

	8.23 IX^8 **22-111-22**	8.24 **11-222-11**	8.25 Mode 6 **1221-122	8.26 **1212-212	8.27 11-21212	8.28 Mode 2 **1212121**	8.29 [Z15] 111-2112	
1	**115 [4]	IIIm^4 [8]	Vm^4 [5]	IIIM^4 [8]	133	X^4	1131	1
2	**22122**	VI^6	VI^6 [1]	SPVIIM [6]	SPVIIm	X^4 [1]	1222	2
3	SPVIIM [11]	IIIM^4 [6]	X^4	**12321**	12123 [1]	**1212121**	1333 [2]	3
4	VIIm^4	**221122** [4]	VI^6 [1]	SPIVm [11]	2213 [8]	X^4	4411 [5]	4
5	**212-212** [4]	IVM^4 [4]	Vm^4	**22122** [11]	2321	X^4 [1]	2311 [7]	5
6	**5** [6]	VI	122	**3**	IIm [1]	IIm^4	IIm	6
I 1-1	**1** [4]	**4** [8]	**6** [5]	[11]	[0]	--	IVm	I 1-1
IIm 1-2	IX [6]	**4** [9]	**6**	XIM [8]	X [1]	X^4	VIIIm	IIm 1-2
IIM 2-1	IX [4]	**4** [6]	**6** [3]	XIm [6]	XIM [10]	X^4 [1]	**3**	IIM 2-1
IIIm 1-3	[5]	IIIm [8]	**6** [5]	IIIM [8]	IIIm	X^4	IIIm [1]	IIIm 1-3
IIIM 3-1	[2]	IIIM [6]	**6** [3]	IIIm [8]	X [10]	X^4	VM [9]	IIIM 3-1
IVm 1-4	IX [6]	IVm [8]	**6**	XIm [8]	**5** [7]	--	I	IVm 1-4
IVM 4-1	IX [2]	IVM [4]	**6** [1]	XIM [4]	**4** [8]	--	IVM [9]	IVM 4-1
Vm 1-5	Vm [5]	**4** [8]	Vm^4 [5]	**3**	X [1]	X^4	Vm	Vm 1-5
VM 5-1	VM[6]	**4** [4]	Vm^4 [6]	**3** [3]	VM [2]	X^4 [1]	231 [7]	VM 5-1
VI 2-2	VIIm^4	VI^6	VI^6 [1]	IX [11]	VI [8]	--	VI [1]	VI 2-2
VIIm 2-3	SPVIIM [9]	**4** [4]	**6** [1]	VIIm^4 [11]	233	X^4 [1]	VM [7]	VIIm 2-3
VIIM 3-2	SPVIIM [11]	**4** [9]	**6**	VIIm^4 [1]	VIIM [2]	X^4 [1]	VIIM [9]	VIIM 3-2
VIIIm 2-4	**5**	VI^6	VI^6 [1]	**3** [6]	VIIIm [8]	X^4 [1]	124	VIIIm 2-4
VIIIM 4-2	**5**	VI^6	VI^6 [1]	**3** [9]	233 [8]	X^4	VIIIM [9]	VIIIM 4-2
IX 5-5	**22122** [2]	**4** [4]	**6** [1]	X^4	IX [2]	--	IX [9]	IX 5-5
X 3-3	**5** [6]	**4** [6]	X^4	SPVIIM [8]	3331 [4]	**1212121**	X^4	X 3-3
XIm 3-4	VIIm^4 [9]	XIm [6]	**6**	214 [6]	233 [5]	X^4	VIIIm	XIm 3-4
XIM 4-3	VIIm^4	XIM [2]	**6** [5]	412 [4]	IIM [10]	X^4	**3** [2]	XIM 4-3
XII 4-4	--	**2**	**2** [1]	[0]	[0]	--	[1]	XII 4-4

	9.1 I^9 **1111-1111**	9.2 1111-1112	9.3 1111-1121	9.4 1111-1211	9.5 1111-2111	9.6 **2111-1112**	
1	I^8	I^7	111-113	111-131	111-311	I^6 [2]	1
2	I^7	111-112	11112	11122	11221	**1122-221 [4]	2
3	I^6	331-111 [6]	133-111 [5]	31121 [9]	12123	**21112**	3
4	**11114	31112 [9]	1313-11 [4]	11-3131 [3]	13211 [8]	**221122** [10]	4
5	**41114** [8]	11232	13121 [8]	113-113 [2]	111-311 [1]	**22122** [10]	5
6	I	IIm	IIM	I [1]	I^4	VI	6
I 1-1	I^7	I^6	I^5	1114	1141	I^5 [2]	I 1-1
IIm 1-2	I^6	11112	1112	112	123	1112 [2]	IIm 1-2
IIM 2-1	I^6	I^5	1113	113	132	2111	IIM 2-1
IIIm 1-3	I^5	1112	11213	SPIVm [3]	421 [8]	112 [2]	IIIm 1-3
IIIM 3-1	I^5	3111 [9]	SPIIIM [9]	3131 [9]	331 [9]	211	IIIM 3-1
IVm 1-4	1114	112	SPIVm [3]	21131	1141 [1]	122 [2]	IVm 1-4
IVM 4-1	4111 [8]	311 [9]	1313 [8]	SPIIIm [8]	1141 [2]	221 [10]	IVM 4-1
Vm 1-5	1141	123	421 [8]	1141 [1]	111-311	VI [2]	Vm 1-5
VM 5-1	1411 [7]	231 [7]	133 [8]	1411 [2]	**11411** [1]	VI [10]	VM 5-1
VI 2-2	I^5	1112	112	122	VI	**221122** [10]	VI 2-2
VIIm 2-3	I^4	1123	IIm^4	214	123 [1]	2212 [10]	VIIm 2-3
VIIM 3-2	I^4	3112 [9]	312 [9]	322 [9]	421 [9]	2122	VIIM 3-2
VIIIm 2-4	114	124	213	114 [1]	1141	VI^6	VIIIm 2-4
VIIIM 4-2	411 [8]	312 [9]	132 [8]	142 [8]	1321 [8]	VI^6	VIIIM 4-2
IX 5-5	IVm^4 [2]	$VIIm^4$ [2]	IVM [3]	1132 [2]	1141 [2]	SPVIIM [7]	IX 5-5
X 3-3	I	3331 [3]	1333 [2]	IVM [9]	3331 [3]	VI	X 3-3
XIm 3-4	IVm	332 [6]	1313 [1]	SPIIIM [9]	331 [6]	322	XIm 3-4
XIM 4-3	IVM [8]	$VIIM^4$ [9]	SPIIIm [1]	3131 [5]	132 [8]	223 [8]	XIM 4-3
XII 4-4	[0]	[1]	**1**	**1**	[0]	**2**	XII 4-4

	9.7 1111-1212	9.8 1111-2112	9.9 IX^9 **1112-2111**	9.10 **1211-1121**	9.11 1112-1121	9.12 Mode 3 **1121-1211**	
1	11113	11131	**11311** [7]	**31113**	11313	**13131**	**1**
2	221112 [8]	**221122** [8]	**212-212** [5]	**21123 [1]	21222 [10]	VI^6	**2**
3	211212 [10]	32121 [7]	**22122** [5]	**1212121**	121-221 [6]	**13131**	**3**
4	22121 [8]	**221122** [10]	**31132 [5]	**12213	1313-11 [5]	**1121-1211**	**4**
5	**212-212**	12311 [7]	**22-111-22** [3]	**12321** [4]	221-211 [5]	**13131**	**5**
6	IIm [1]	112	I [1]	IIm^4	IIm	VI	**6**
I **1-1**	I^4	114	IVm^4 [7]	I [3]	IVm	XII	I **1-1**
IIm 1-2	1123	124	VIIm^4 [7]	3331 [6]	332 [6]	XII [1]	IIm 1-2
IIM 2-1	2113 [10]	213 [10]	VIIm^4 [5]	1333 [3]	VIIM^4 [7]	XII [2]	IIM 2-1
IIIm 1-3	IIm^4	213	IVm [8]	1333 [5]	1313 [1]	**13131**	IIIm 1-3
IIIM 3-1	213 [10]	231 [10]	IVM [5]	3331 [3]	SPIIIm [2]	**13131** [1]	IIIM 3-1
IVm 1-4	214	114 [1]	1132 [7]	IVM	SPIIIM [9]	**13131**	IVm 1-4
IVM 4-1	223 [8]	241 [8]	2311 [3]	IVm	1313 [9]	**13131**	IVM 4-1
Vm 1-5	123 [1]	1141	1141 [7]	3331 [6]	331 [6]	XII	Vm 1-5
VM 5-1	124 [7]	1231 [7]	1411 [2]	1333	231 [7]	XII	VM 5-1
VI **2-2**	2212 [8]	VI^6	SPVIIM [10]	VI [1]	322 [10]	VI^6	VI **2-2**
VIIm 2-3	22212 [8]	231 [8]	**22122** [3]	3331 [7]	2321 [5]	XII	VIIm 2-3
VIIM 3-2	SPVIIM [7]	332 [7]	**22122** [5]	1333	2122 [7]	XII [1]	VIIM 3-2
VIIIm 2-4	231 [8]	**221122** [8]	241 [1]	1333 [3]	124	VI^6	VIIIm 2-4
VIIIM 4-2	233 [8]	VI^6	142 [8]	3331 [3]	233 [1]	VI^6	VIIIM 4-2
IX **5-5**	**22122** [10]	241 [8]	**212-212** [7]	IX [7]	SPVIIM [2]	XII	IX **5-5**
X **3-3**	3331 [4]	1333	IX [9]	**1212121**	3331 [9]	XII [2]	X **3-3**
XIm 3-4	2312 [5]	124	SPVIIM [7]	3331	13221 [6]	**13131** [1]	XIm 3-4
XIM 4-3	2212 [8]	332	SPVIIM [10]	1333 [5]	SPIVm [5]	**13131** [1]	XIM 4-3
XII **4-4**	[0]	**2**	[1]	[1]	**1** [1]	I	XII **4-4**

	10.1 I^{10} **111-111-111**	10.2 **111-1221-11	10.3 **111-212-111**	10.4 **111-2112-11	10.5 IX^{10} **112-111-211**	10.6 Mode 7 **1111-2-1111**	
1	I^9	I^8 [8]	**1113-311 [11]	**311-1-113** [6]	**1131-131 [11]	**111-3-111**	1
2	I^8	**2111-1112** [6]	**211-1-112** [6]	**1122-221 [11]	**22-111-22** [5]	**1221-122 [1]	2
3	**1113-311 [3]	**211-1-112** [6]	**1211-1121** [5]	**12-111-21** [6]	**1212-212 [10]	**1212121**	3
4	**113-1-311** [3]	**1122-221 [10]	**12-111-21** [5]	**1121-1211** [1]	**11-212-11** [5]	**1221-122 [9]	4
5	**1131-131 [2]	**122-1-221** [10]	**122-1-221**	**11-212-11** [5]	**111-22-111** [4]	**111-3-111** [1]	5
6	I^4	211	IIM^4	122	$IIIm^4$	I^5	6
I 1-1	I^8	I^7 [8]	I^6 [8]	**11114 [9]	**14141** [11]	**11411**	I 1-1
IIm 1-2	I^7	111-112 [8]	33-1111 [2]	31112 [6]	11232 [9]	12312	IIm 1-2
IIM 2-1	I^7	211-111 [6]	1111-33 [8]	21113 [7]	23211 [2]	21321 [4]	IIM 2-1
IIIm 1-3	111-113	11112 [8]	133-111 [1]	1313-11 [1]	13121 [5]	21321	IIIm 1-3
IIIM 3-1	311-111 [9]	21111 [6]	111-331 [8]	11-3131 [9]	12131 [6]	12312 [3]	IIIM 3-1
IVm 1-4	111-131	11122 [8]	31121 [5]	11-3131	113-113 [11]	**11411** [1]	IVm 1-4
IVM 4-1	131-111 [8]	22111 [4]	12113 [5]	1313-11 [9]	311-311 [9]	**11411** [2]	IVM 4-1
Vm 1-5	111-311	11221 [8]	12123 [8]	13211 [5]	111-311 [10]	**111-3-111**	Vm 1-5
VM 5-1	113-111 [7]	12211 [3]	32121	11231 [5]	113-111 [11]	**111-3-111** [1]	VM 5-1
VI 2-2	I^6	**1122-221 [10]	**21112** [6]	**221122** [5]	**22122** [5]	VI^6	VI 2-2
VIIm 2-3	11113	221-112 [4]	211-212 [6]	22121 [5]	**212-212** [9]	12312 [1]	VIIm 2-3
VIIM 3-2	31111 [9]	211-122 [6]	212-112 [3]	12122 [6]	**212-212** [11]	21321 [1]	VIIM 3-2
VIIIm 2-4	11131	**221122** [4]	32121 [3]	**221-122** [7]	12311 [4]	**1221-122 [1]	VIIIm 2-4
VIIIM 4-2	13111 [8]	**221122** [4]	12123 [5]	**221-122** [1]	11321 [5]	**1221-122 [3]	VIIIM4-2
IX 5-5	**11311** [2]	**212-212** [8]	**22122** [8]	**13223 [1]	**22-111-22** [7]	**11411** [2]	IX 5-5
X 3-3	**31113** [9]	**12332 [9]	**2121212**	**21331 [9]	**12321** [10]	**1212121**	X 3-3
XIm 3-4	11313	21222 [6]	121-221 [2]	1313-11 [2]	221-211 [2]	12312	XIm 3-4
XIM 4-3	31311 [5]	22212 [2]	122-121 [5]	11-3131 [5]	112-122 [5]	21321	XIM 4-3
XII 4-4	**1**	**2**	**1** [1]	I [1]	**1** [1]	**2**	XII 4-4

Appendix I:
Some Global Chromatic Structures

The chart opposite illustrates how the intervallic prime forms can clarify the more global structures of the chromatic system. EV (equal value) signifies intervals of equal value or size, with the superscript number showing the number of such intervals, e.g., EV^2 means 2 equal intervals, EV^3 means 3 equal intervals, etc. S signifies a small interval, L a large interval, and M a medium interval (these sizes are relative, naturally: e.g., a 'medium'-sized interval, if it is combined *only* with a 'small' interval, becomes a 'large' interval, in relation to that small interval). Combinations containing these various intervals are all permutated, e.g. SL has one other permutation, namely, LS; SLS has two other permutations, SSL and LSS, etc. (Bracketed groups in smaller print appear elsewhere in the table, in the same or in a different intervallic guise.)

If EV is taken also to signify equal *time* intervals or values, with S being taken also to signify a *short* time interval, and L to signify a *long* time interval, then the chart also shows at the same time the six primary rhythmic cells (EV^2, EV^3, SL, SSL, LLS, SML) and their permutations and expansions, so it becomes a rhythmic table as well. Thus the most fundamental time divisions and groupings are also implied in the structure of the chromatic system.

Intervallic Structure of the Dyads, Triads, Tetrads & Pentads

6 DYADS

1 | 2 | 3 | 4 | 5 | 6

12 TRIADS

EV2 SL -------->

1-1 1-2 1-3 1-4 1-5

2-2 2-3 2-4 (2-5)

3-3 3-4

4-4

5-5

29 TETRADS

EV3 **111** SSL ---> 112 113 114 **115

LSL SLS SLL SML -->

212 **121** 122 (123) (124) ---------> SML

313 **131** 133 (134)

414 **141** 144

(515)→ =**151** (**155)

123 132 213

124 142 214

EV3 **222** SSL----> 223 **224

LSL SLS SLL---->

323 **232** 233 (**244)

(424)--> =**242**

EV3 **333**

38 PENTADS

SSLS--> 1121 1131 1141

EV4 **1111** SSSL--> 1112 1113 1114

SLSL LSSL SLLS SSLL--> 1122 1133 1144

1212 **2112** **1221**

(2113)

SLLL--> 1222 1333

1313 **3113** **1331**

LSLL--> 2122 3133

1414 **4114** **1441**

EV4 **2222**

SLSL LSSL SLLS

(2323)→ **3223** **2332**

3434

SSML

MSSL 2113

----> <----

1 123 (1 321)

1 132 1 231

1 213 1 312

SMML

----> ---->

2 123 123 2

2 132 132 2

2 213 (213 2)

Appendix II:
The Hour-Groups

The main chart in this appendix shows all of the hour-groups, from triads to 12-note groups, for each of the twelve hours.

An hour-group, as stated earlier, is any intervallic form which can be interpreted overall as belonging in a *single* hour.

A number of chromatic groups whose IPF is *not* an hour-group can nevertheless be permutated so as to form an hour-group, or even several different hour-groups. Moreover, the same can be true of chromatic groups whose IPF *is* already an hour-group—these can form other hour-groups as well.

Symmetrical Hours

For the simple symmetrical hours—the first, sixth, tenth and twelfth hours—the constitution of their hour-groups is perfectly easy, since these hours always have a single interval only, which is simply repeated (stacked) to form the larger hour-groups, themselves also always symmetrical (apart from the exceptional ninth-hour triad and tetrad). Thus these symmetrical hours have a much smaller number of hour-groups than most of the others (the twelfth hour has only one member), and their configurations are so simple that they require no discussion.

Asymmetrical Hours

Two of the asymmetrical hours (i.e., those hours containing two different intervals) also have a very small number of members, namely, the fifth and eighth hours. As with the symmetrical tenth hour, this is because their single symmetrical tetrad (in which the minor form = the major form) already produces a subscale formation beyond which no further hour-groups can be formed, apart from two 'impossible' symmetrical pentads.

One can easily enough familiarise oneself with all of the above hours, which have relatively few members. Their individual 'climates' or colours are particularly strong—and therefore memorable, in the same sort of way that the primary colours, for a painter, are memorable. Moreover, in tone-clock terms they are also powerful steering hours, and can have an important part to play at deeper structural levels.

For the remaining asymmetrical hours—the second, third, fourth, seventh and eleventh hours, as well as the asymmetrical form of the ninth hour—some further explanation is necessary. These hours have a large and rich array of hour-groups, formed in line with the following conditions (which also apply to the asymmetrical fifth and eighth hours, for that matter).

In these hours, no hour-group can *begin or end* with a *repeated* interval. This is because a repeated interval always signifies a symmetrical triad in a symmetrical hour—and we are dealing here with the *asymmetrical* hours (an hour-group, we recall, must be able to be interpreted in a *single* chromatic hour). A repeated interval, however, *can* appear *within* an hour-group (though it can only be repeated once), because in this context, although it still signifies a symmetrical triad of a different hour, it can still *also* be interpreted in the original asymmetrical hour.

Take the second-hour group SP IIm, for example: **1221**. The two wholetones in the middle of this prime form certainly constitute a nuclear triad in the sixth hour, and not a second-hour triad. However, the group as a whole can *also* be interpreted as a minor second-hour triad followed by a major second-hour triad, with the two triads connected by pivoting (the last note of the first triad becomes the first note of the second triad)—hence the whole thing can be read in a single hour. This would not be so if there were *three* wholetones in the middle, say (e.g., **12221**), or if there were two semitones at the *end* or at the *beginning* of the group (e.g., 1211, say, or 11221). The latter intervallic forms can no longer be interpreted in a single hour, since the sixth hour and the first hour, respectively, are unavoidably present, and there is no possibility of an alternative interpretation in a single hour—thus they are not hour-groups. (There is still the possibility, however, that the notes of these groups, if *permutated*, might form a viable hour-group: so we can look them up in the IPF chart, and there we shall see that, as it happens, one of these groups, **12221**, although it is not an hour-group in this compact intervallic form, does form two oedipus hour-groups, namely, IVM^6 **41414** and XIM^6 **43434**.)

The above logical limitations satisfactorily restrict the number of possible asymmetrical hour-group configurations that can theoretically be formed. This number is further restricted, in any given hour, by the fact that certain configurations produce note-repetitions. Such configurations are excluded from the present hour-group chart, unless they are 'impossible symmetries'. Adopting the symbols from Appendix I ('S' = a small interval, 'L' = a large interval), Figure 1.3 shows all the possible configurations, from the asymmetrical triad through to the 11-note hour-configurations. (Although there is technically only *one* 11-note chromatic group as such, this group will form seven different 11-note hour-groups without note-repetitions.) The 12-note configurations are not included in Figure 1.3 because they would take up an inordinate amount of space, and there are in any case only four of these that are possible in the asymmetrical hours. Bold type indicates a symmetrical configuration. For the remaining, asymmetrical configurations, the 'inversion' is seen by reading the sequence backwards (just as in the IPF chart). By reason of note-repetitions, no single hour ever contains all of these configurations.

Configurations of the Asymmetrical Hours

	MINOR FORM	MAJOR FORM	No
asymmetrical triad	SL	inversion	1
symmetrical tetrad	**SLS**	**LSL**	2
oedipus pentad	SLSL	inversion	3
symmetrical pentad	**SLLS**	**LSSL**	4
oedipus hexad	**SLSLS**	**LSLSL**	5
asymmetrical hexad	SLSSL	LSLLS	6
oedipus heptad	SLSLSL	inversion	7
gemini	**SLS-SLS**	**LSL-LSL**	8
asymmetrical heptad	SLLSSL	inversion	9
" "	SLSLLS	LSLSSL	10
oedipus octad	**SLSLSLS**	**LSLSLSL**	11
symmetrical octad	**SLLSLLS**	**LSSLSSL**	12
asymmetrical octad	SLS-SLSL	LSL-LSLS	13
" "	SLS-SLLS	LSL-LSSL	14
" "	SLS-LSSL	LSL-SLLS	15
greater gemini	**SLLS-SLLS**	**LSSL-LSSL**	16
oedipus nonad	SLSL-SLSL	inversion	17
oedipus twin	**SLSL-LSLS**	**LSLS-SLSL**	18
asymmetrical nonad	SLLS-LSSL	inversion	19
" "	SLSL-SLLS	LSLS-LSSL	20
" "	SLSL-LSSL	LSLS-SLLS	21
" "	SLS-SLSLS	LSL-LSLSL	22
" "	SLLS-LLSL	LSSL-SSLS	23
" "	SLSS-LLSL	inversion	24
gemini triplet	**SLS-SLS-SLS**	**LSL-LSL-LSL**	25
oedipus decad	**SLSLSLSLS**	**LSLSLSLSL**	26
symmetrical decad	**SLLS-L-SLLS**	**LSSL-S-LSSL**	27
asymmetrical decad	SLS-SLSLSL	LSL-LSLSLS	28
" "	SLSL-LSLSL	LSLS-SLSLS	29
" "	SLSL-LSSLS	LSLS-SLLSL	30
" "	SLSL-LSLLS	LSLS-SLSSL	31
" "	SLLS-SLSLS	LSSL-LSLSL	32
" "	SLLS-LSLSL	LSSL-SLSLS	33
" "	SLLS-LSSLS	LSSL-SLLSL	34
" "	SLLS-LLSSL	LSSL-SSLLS	35
" "	SLLS-SLLSL	LSSL-LSSLS	36

(Fig. 1.3.....)

	MINOR FORM	MAJOR FORM	No
symmetrical 11-note	**SLSLS-SLSLS**	**LSLSL-LSLSL**	37
oedipus 11-note	SLSLS-LSLSL	inversion	38
asymmetrical 11-note	SLSLS-SLSSL	LSLSL-LSLLS	39
" "	SLSLS-LSLLS	LSLSL-SLSSL	40
" "	SLSLS-LLSLS	LSLSL-SSLSL	41
" "	SLSLS-LLSSL	LSLSL-SSLLS	42
" "	SLSLS-LSSLS	LSLSL-SLLSL	43
" "	SLSLS-SLLSL	LSLSL-LSSLS	44
symmetrical "	**SLSSL-LSSLS**	**LSLLS-SLLSL**	45
asymmetrical "	SLSSL-SLSSL	LSLLS-LSLLS	46
" "	SLSSL-LSLLS	LSLLS-SLSSL	47
" "	SLSSL-SSLSL	LSLLS-LLSLS	48
" "	SLSSL-SSLLS	LSLLS-LLSSL	49
" "	SLSSL-SLLSL	inversion	50
symmetrical "	**SLLSL-LSLLS**	**LSSLS-SLSSL**	51
asymmetrical "	SLLSL-SLSSL	inversion	52
" "	SLLSL-SSLSL	LSSLS-LLSLS	53
" "	SLLSL-SSLLS	LSSLS-LLSSL	54
" "	SLSLL-SSLSL	inversion	55
" "	SLSLL-SSLLS	LSLSS-LLSSL	56
" "	SLLSS-LLSSL	inversion	57

Fig. 1.3

Note that if any of the configurations in Figure 1.3 turns out, in a given hour, to be an 'impossible symmetry', then it will move back to the next smallest class. Thus the two greater-gemini configurations, for example, which are listed (in the abstract, so to speak) as nonads, turn out in the second hour to be impossible symmetries (both contained within the notes of a single chromatic group, moreover, i.e., Mode 6). The main hour-group chart therefore lists them in the second hour under the octad, and not the nonad, heading.

Observe, too, how a configuration such as No. 57, for example, requires only the dislodgement of the first note (removing the first interval) in order to 'fall apart', so to speak, into two pairs of triads in two different symmetrical hours, that is to say, the 'LL' hour and the 'SS' hour. There are numerous comparable examples. In a sense, the hour-groups are like musical 'atoms' and 'molecules': they may be made to move around and 'bump into' each other, lose a note here or there and become transformed, or decompose into smaller groups, and so on.

I find it continually fascinating, both within the chromatic system and beyond it (for Figure 1.3 has other possible applications—once again, it could apply to time intervals, for instance), to see how such a variegated array of individual structures can so quickly emerge from a small number of elements, simply by permutating, combining and regrouping them (in this case, under certain specified conditions). Mathematics seems to be inherently 'creative'; certainly one can perceive a sort of musical 'entelechy' at work in the chromatic system itself, long before any composers have got their hands on it. In fact, for anyone desiring to understand in principle how entelechy itself can come about, I should think the chromatic system and its groups provide a perfect illustration.

We have discussed the 'multiple nature' aspect in which a single chromatic group can sometimes form a number of hour-groups in different hours. As can often be seen in the following hour-group chart, this multiple nature extends even further, for it is found

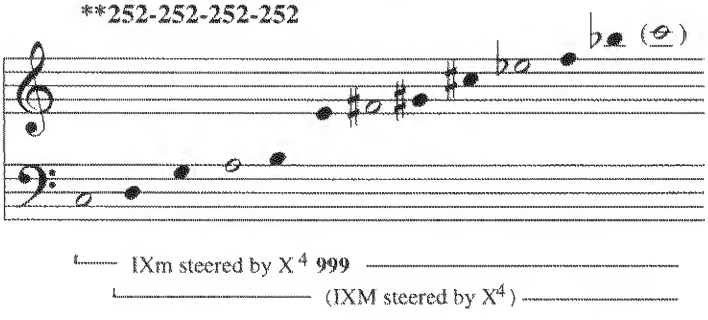
also *within* individual hours. Thus the second hour, for example, contains all ten of the possible octad configurations shown above (not counting the impossible symmetries mentioned a moment ago). These ten configurations are not formed, however, from ten different chromatic groups but from only *five* different groups. In other words, a single chromatic group will sometimes form a number of different hour-groups in the *same* hour, as well as, or instead of, in *different* hours (this property is specifically foreshadowed in the ninth-hour triad).

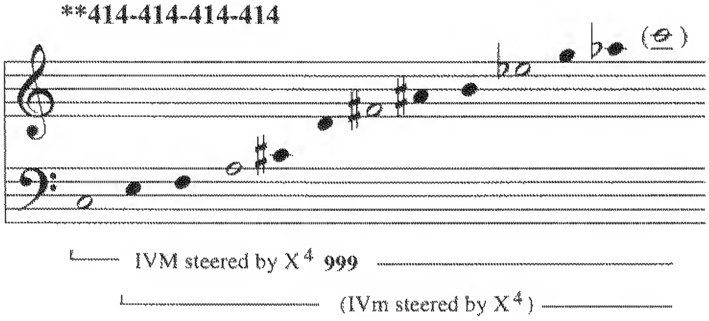
With one exception (the fourth-hour oedipus subscale, **17171**) the intervals contained in the hour-group chart are no larger than a perfect 4th. All of the configurations shown can also be re-formed using the octave inversions of the interval concerned. (Their range, of course, will then be considerably greater, and will in some cases exceed what is musically possible.)

Roughly half of the 223 chromatic groups are either hour-groups already (i.e., in their prime forms) or else they will form hour-groups. That is to say, there are eighty or so tone-clock groups (the simplest hour-groups), including all the triads, symmetrical tetrads, symmetrical pentads, and oedipus groups, and all of the simple symmetrical groups in the first, sixth and ninth hours. All of these are IPFs or alternative IPFs. There are in addition a number of IPFs or alternative IPFs which are larger hour-groups without specific roman numeral names (such as geminis, etc., and a few second-hour groups); then there are still other chromatic groups whose actual IPFs are *not* hour-groups, but whose notes will nevertheless form a variety of (mostly larger) hour-groups.

In addition to the two 12-note hour-groups in the symmetrical hours (i.e., I^{12} and IX^{12}), there are four more 12-note hour-groups, as I have said, in the asymmetrical hours and the asymmetrical ninth hour. In the ninth hour there is **252-252-252-25; in the eleventh hour there is **4334-434-4334**; and in the fourth hour there is 4114-1441-141 and **14-414-414-414. The first ninth-hour example and the last example are actually 13-note impossible symmetries, namely ****252-252-252-252** and ****414-414-414-414** (i.e., both 'gemini quadruplets'). If we look at these last two configuration more closely (writing them out in notes), we shall see that they are actually two of Schat's 12-note toneclock tonalities: triad IX(m or M) and triad IV (m or M), respectively, steered by X^4 spaced as **999**—i.e., each now expressed as a giant gemini scale. These two configurations are further gateways into the same new area of the chromatic system mentioned earlier in the text, which must await a later chromatic map.

Giant Geminis





Hour-Groups

I

I
I^4
I^5
I^6
I^7
I^8
I^9
I^{10}
I^{11}
I^{12}

II

II	PC Set
Triad	
II(m,M)1-2........................	3.2
Tetrads	
(ST)IIm^4 **121**.....................	4.3
(ST)IIM^4 **212**.....................	4.10
Pentads	
SPIIm **1221**......................	5.Z12
SPIIM **2112**......................	5.8
(oedipus)II^5(m, M)1212..........	5.10
Hexads	
(oedipus)IIm^6**12121**.............	6.Z13
(oedipus)IIM^6**21212**............	6.Z23
12112(m)..........................	6.Z10
21221(M)..........................	6.Z25
Heptads	
(oedipus)II^7(m, M) 121212......	7.31
(gemini)**121-121**(m)............	7.Z37
(gemini)**212-212**(M)............	7.35
122112(m, M)....................	7.28
121221(m).........................	7.32
212112(M).........................	7.25
Octads	
(oedipus) II^8(m = M) **1212121=2121212** *second-hour subscale*	8.28 (Mode2)
1221221(m)......................	8.23
2112112(M)......................	8.24
(greater geminis) (m = M) ****1221-1221** = ****2112-2112**.....	8.25 (Mode6)
(oedipus twins) (m = M) ****1212-2121** = ****2121-1212** = 121-1212(m) = 212-2121......	8.26
121-1221(m) = 212-2112(M) = 121-2112(m) = 212-1221(M....	8.27
Nonads	
121-12121(m).....................	9.11
212-21212(M).....................	9.7
21121121 = gemini triplet ****121-121-121(m)......................	9.12 (Mode3)
Decads	
2112-1-2112........................	10.3
211211212 (M)..................	10.4

III

III	PC Set
Triad	
III(m,M)1-3....................	3.3
Tetrads	
(ST)IIIm^4 **131**.................	4.7
(ST)IIIM^4 **313**................	4.17
Pentads	
SPIIIm **1331**..................	5.22
SPIIIM **3113**..................	5.Z37
(oedipus)III^5(m, M) 1313.....	5.21
Hexads	
(oedipus)III^6(m = M) **13131=31313** *third-hour subscale*	6.20
13113(m)=31331(M).........	6.Z44
Heptads	
(gemini)**131-131**(m)..........	7.22
(gemini)**313-313**(M)..........	7.Z17
Octads	
1331331(m)..................	8.7
3113113(M)..................	8.20
Nonads	
13313313(m) = 31131131(M)..................	9.4
Decads	
gemini triplet **131-131-131**(m)	10.1
gemini triplet **313-313-313** (M)	10.5

11-note Group
1331331331

Hour-Groups

IV

	PC Set
Triad	
IV(m,M)1-4	3.4
Tetrads	
(ST)IVm^4**141**......................	4.8
(ST)IVM^4**414**......................	4.20
Pentads	
SPIVm**1441**	5.Z17
SPIVM**4114**	5.15
(oedipus)IV^5(m, M) 1414	5.20
Hexads	
(oedipus)IVm^6**14141**............	6.Z38
(oedipus)IVM^6**41414**...........	6.Z26
(gemini)****141-141** *fourth-hour gemini subscale*	6.7
(17171=71717) *fourth-hour oedipus subscale)*....	6.20
14114(m)	6.7
41441(M)	6.14
Heptads	
(oedipus)IV^7(m,M) 141414......	7.14
(gemini)**414-414**(M)............	7.Z17
144114(m,M)......................	7.9
141441(m).........................	7.11
414114(M).........................	7.7
Octads	
(oedipus)IVm^8**1414141**.........	8.6
(oedipus)IVM^8**4141414**.........	8.23
1441441(m).......................	8.20
1414-414(m).......................	8.14
141-1441(m).......................	8.5
414-4114(M).......................	8.11
414-1441(M).......................	8.22
Nonads	
(greater gemini) **1441-1441**(m)= 1414-4114= ****1441-4-1441**..........	9.6
(oedipus)IV^9(m,M)1414-1414 = (oedipus twin) **1414-4141**(m) = 1414-1441 = 414-41414.........	9.9
1441-1414 =1411-4414 = 1441-4114 = 14414414...........	9.4
Decads	
(oedipus)IVm^{10}1414-14141......	10.5
gemini triplet **414-414-414** (M).	10.1
141-1441-41.......................	10.4

(IV cont.)

	PC Set
11-note Groups	
1441441441 = 41141-44114	
12-note Groups	
****414-414-414-414** 4114-1441-141	

V

	PC Set
Triad	
V(m,M)1-5..................	3.5
Tetrads	
(ST)V^4(m=M) **151 = 515** *fifth-hour subscale*........	4.9
"SP Vm" ****1551** = "SP VM" ****5115**.........	4.6

VI

VI
VI^4
VI^5
VI^6

Hour-Groups

VII

	PC Set
Triad	
VII(m,M)2-3	3.7
Tetrads	
(ST)VIIm⁴**232**	4.23
(ST)VIIM⁴**323**	4.26
Pentads	
SPVIIm **2332**	5.34
SPVIIM **3223** = (oedipus)VII⁵(m,M) **2323 = ****23232**	5.35
Hexads	
(oedipus)VIIM⁶**32323**	6.32
32332(M)	6.Z46
Heptads	
(gemini)**323-323**(M)	7.Z17
232332(m)	7.23
Octads	
2332332(m)	8.24
2323-323(m)	8.14
323-2332(M)	8.22
Nonads	
(oedipus twin) **2323-3232**(m)	9.9
23323323(m) = (gemini triplet) ****323-323-323**	9.12
323-32323(M) = 323-23323(M)	9.11
Decads	
2332-3-2332	10.3
2323-32323	10.5
11-note Group	
32323-32323	

VIII

	PC Set
Triad	
VIII(m,M)2-4	3.8
Tetrads	
(ST)VIII⁴(m = M) **242 = 424** *eighth-hour subscale*	4.25
"SP VIIIm" ****2442** = "SP VIIIM" ****4224**	4.24

IX

	PC Set
Triad	
IX(m, M, symmetrical) 2-5, **5-5**	3.9
Tetrad	
IX⁴ **555** = IXm⁴**252**	4.23
Pentad	
IX⁵ **5555** = SPIXM **5225**	5.35
Hexad	
IX⁶ **55555** = 25225	6.32
Heptad	
IX⁷ **555-555** = (gemini)**252-252**(m)	7.35
Octad	
IX⁸ **555-5-555** = **5225225**	8.23
Nonad	
IX⁹ **5555-5555** = 252-25225(m)	9.9
Decad	
IX¹⁰ **5555-5-5555** = (gemini triplet) **252-252-252**	10.5
11-note Group	
IX¹¹ **55555-55555 = 5225225225**	
12-note Group	
IX¹² **55555-5-55555 =**252-252-252-252**	

Hour-groups

X	XI	PC Set	(XI cont.)	XII
X	**Triad**		**12-note Group**	XII
X^4	XI(m,M)3-4	3.11	**4334-434-4334**	*twelfth-*
tenth-				*hour*
hour	**Tetrads**			*subscale*
subscale	(ST)XIm^4 **343**	4.26		
	(ST)XIM^4 **434**	4.20		
	Pentads			
	SPXIm **3443**	5.Z17		
	SPXIM **4334**	5.34		
	(oedipus)XI^5(m,M) 3434...........	5.27		
	Hexads			
	(oedipus)XIm^6 **34343**	6.32		
	(oedipus)XIM^6 **43434**	6.Z26		
	35353=53535 *eleventh-hour oedipus subscale*	6.20		
	34334(m)..............................	6.33		
	43443(M)..............................	6.Z19		
	Heptads			
	(oedipus)XI^7(m,M) **343434 = **3434343** =			
	(gemini)**343-343**(m).................	7.35		
	(gemini)**434-434**(M)................	7.22		
	344334(m,M)..........................	7.34		
	343443(m)............................	7.32		
	434334(M)............................	7.35		
	Octads			
	(oedipus)XIM^8 **4343434**...........	8.23		
	3443443(m)..........................	8.7		
	3434-434(m)..........................	8.18		
	434-4334(M)..........................	8.27		
	434-3443(M)..........................	8.Z15		
	Nonads			
	(greater gemini)**3443-3443**(m)....	9.6		
	(oedipus twin)**3434-4343**(m)......	9.10		
	434-43434(M)........................	9.5		
	3443-4343(m)........................	9.7		
	34434434(m)..........................	9.4		
	Decads			
	3443-44334 = ****3443443443**.....	10.4		
	gemini triplet **434-434-434** (M)= 3443-43434 = 344344343 = 343434-434............................	10.5		
	****43434-43434** = **3443-4-3443**.......................	10.6		
	11-note Group			
	3434-434343			

Appendix III: Finding Prime Forms

In order to identify and refer easily to each of the chromatic groups, it is a useful convention to select one particular permutation of its notes as the 'main' form, or so-called 'prime form', by which this group can then be listed in the reference chart.

On pp14-15 I have discussed the criteria involved in selecting the present prime forms. These can be summarised as follows:

1. The notes of the group are shown in their most compact rising form, with the smallest intervals first. But in addition:-
2. A symmetrical permutation, where any exists, is preferred to an asymmetrical permutation. If more than one symmetrical permutation is possible, the more compact one is preferred (though other[s] may be listed as alternatives). And also:-
3. A permutation containing fewer interval classes is preferred to one containing a larger number of interval classes.

The above conditions are weighed against one another, so that No. 2 or 3 above may override No. 1, provided that the selected permutation is still reasonably compact. (In such cases, the most compact but less simple form is usually shown as well, in the 'Other Identities' Column.)

Those who are inexperienced (and even those who are not) may sometimes have difficulty locating a prime form. In such cases, first be assured that all of the chromatic groups are indeed represented in the table (so you needn't write to me saying I've missed one out).

To find a prime form, first put the notes of the group concerned into their most compact ascending order, with the smallest intervals first. (Eliminate any note-repetitions that may be present, and eliminate any registrations in different octaves—i.e., bring all the notes together so that they occur in the same octave). Note that with the IPFs there is no need, necessarily, to transpose this note-arrangement so that it begins on the traditional middle C—the prime form can be identified from any transposition of the notes.

Example 1: Let us say we have found for ourselves a group containing the following notes:

g# f# f' c' g# a c d

We eliminate the different octaves and the g# and c note-repetitions, and put the notes into their most compact rising order, with the smallest intervals first, thus:

	f	f#	g#	a	c	d
intervals in semitones:	1	2	1	3	2	

Having done this, however, we are still unable to find 12132 listed as a prime form anywhere in the hexad section of the chart. (We know the group is a hexad because it has only six notes, or five intervals.)

First trouble-shooting tip: Now we check to see whether 12132 is listed anywhere in the 'Other Identities' column (remembering, too, that a group may sometimes be listed here under its inversion, i.e., the backwards form, with the smaller intervals at the end instead of the beginning—in this case 23121). But it isn't.
Second trouble-shooting tip: So now we check to see whether the group has a symmetrical form that we haven't noticed. To discover this, we simply keep on adding the notes of the group, always in their most compact rising order, at the end of what we have already written (this can also be done using a clock-face to represent the twelve chromatic note-positions in order, if you like[1]). Thus:

[1] The use of the clock-face (suggested by Ross Harris) is also a simple way to check that you have actually found the most compact form in the first place, for sometimes you may not have.

(the next octave is implied)

f f# g# a c d f f# g# a c d

intervals in semitones: 1 2 1 3 2 3 1 2 1 3 2

symmetrical

symmetrical

By doing this, we can now see that our group actually has two compact symmetrical forms, **13231** and **31213**, of which the most compact is the first. So now we look for **13231** in the IPF column of the hexad section, and lo, we find that it is the group 6.49 (with the other compact symmetrical form **31213** listed in the 'Other Identities' Column).

Example 2: Suppose the notes of the group (in their most compact rising order, smallest intervals first, etc.) are these: e f a b^{b} c

1 4 1 2

Although this intervallic form is in fact listed (under its inversion) in the 'Other Identities' Column of the pentad section, let us pretend for the moment that it is not there and that we cannot find the prime form for this group in the table. So once again we keep adding the same notes in order at the end, as before:

e f a b^{b} c e f a b^{b} c

1 4 1 2 4 1 4 1 2

This time there is no symmetrical form that we haven't noticed, but we can see that the bracketed permutation above—c, e, f, a, b^{b}, or 4141—contains only *two* interval classes, 4 and 1, whereas every other possible form above will contain *three* interval classes, 4, 2 and 1. Thus 4141 is preferred, because of condition No. 3, even though it is somewhat less compact. This form will be listed in the pentad IPF column under its inversion 1414 (5.20), however, because of the 'smallest intervals first' condition.

Example 3: (This example concerns a group over which there may well be some argument, in which there is certainly a case for the most compact form being taken as an alternative IPF). The notes (most compactly, etc.) are:

b c d e g

1 2 2 3

This is listed in fact (under 'Other Identities') but again we shall pretend we cannot find it. So we write it out again as usual, but can find no concealed symmetries, nor can we see any permutation in which fewer than three interval classes would be involved:

b c d e g b c d e g

1 2 2 3 4 1 2 2 3

So now we look to see whether the notes can be reasonably compactly *rearranged* so as to contain a smaller number of interval classes. This is done by checking out points at which two adjacent intervals can be added in such a way that all five notes are still represented but a smaller number of interval classes is involved, e.g.:

b c d e g b c d e g

1 [2 + 2] 3 4 [1 + 2] 2 3

c e g b d

4 3 4 3

Thus this group is most simply represented as 4343, listed under the inversion 3434 (5.27). There are not many cases in which you will need to do this, however, in order to find an IPF, and as I say there is certainly a fair case here for taking 1223 as an alternative IPF, since 3434 is considerably less compact. We need not worry too much about which of these is 'correct', provided it is clear which chromatic group we are actually talking about.

If you are dealing with a group larger than a hexad, much the easiest way to find the IPF is to work out its smaller chromatic complement first and look that up instead—the chart will tell you (in the Chromatic Complement column) what the original larger group is, or alternatively, you can simply use the PC set numbers, e.g., 8.19 is the complement of

4.19, 7.27 is the complement of 5.27, 9.3 is the complement of 3.3, and so on (this doesn't work with the hexads, though).

Appendix IV:
Schat's 'Zodiac of the Twelve Tonalities'

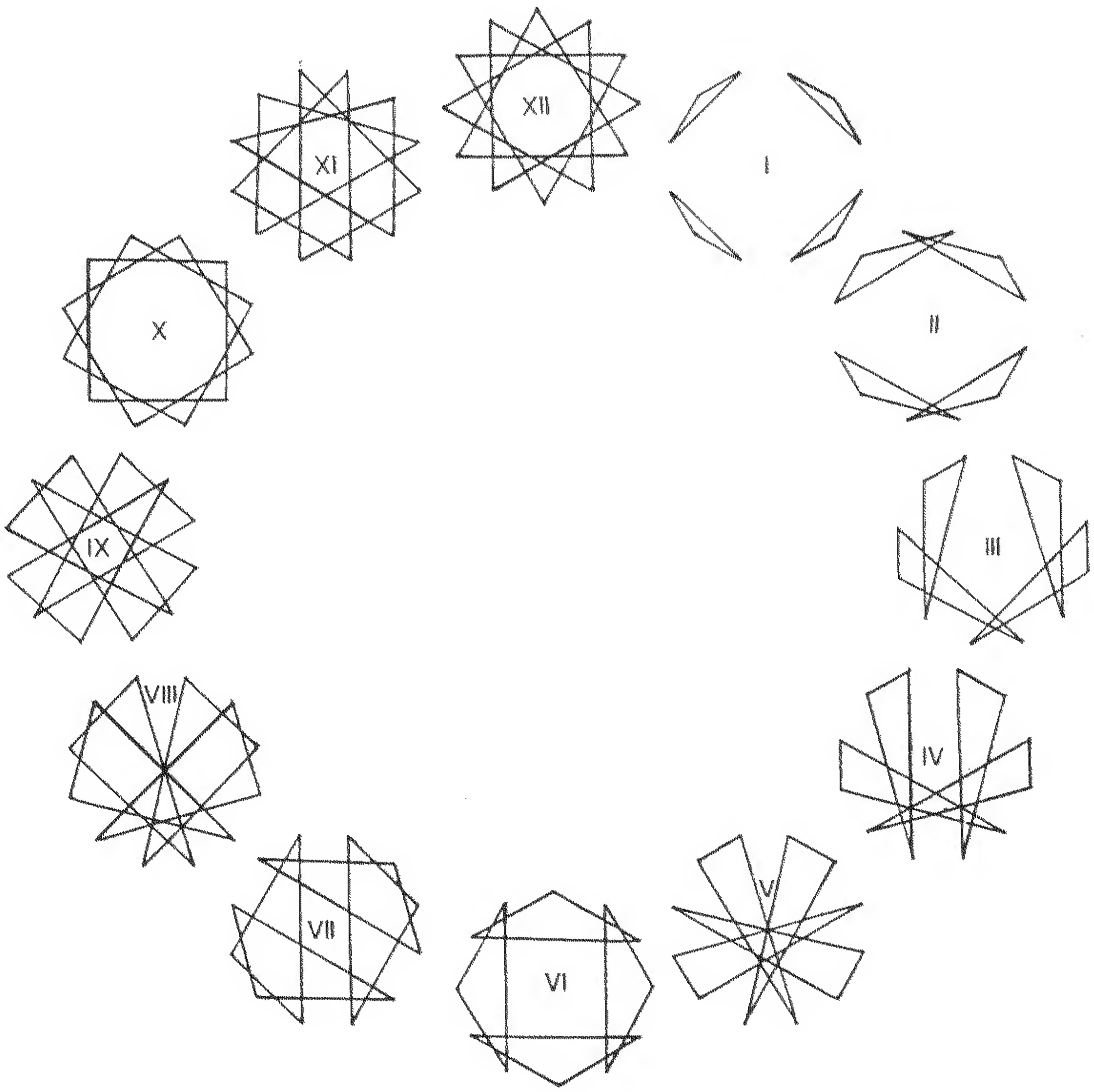
(see Schat [1993], Chapter 6)

Twelve small clock-faces are contained within a larger clock-face. This is a visual representation of chromatic properties shown in the 'TC Steerings' column of the IPF chart, p39, in the triad section (and also shown in stave notation on p132).

At the 'one o'clock' position on each smaller clock-face is *c*, with *c#* at 'two o'clock', *d* at 'three o'clock', and so on through the chromatic scale to *b* at 'twelve o'clock'. The four triangles on each smaller clock-face represent the four chromatic triads of the particular 12-note chromatic tonality shown on that clock-face.

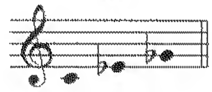
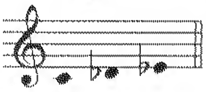
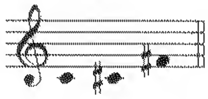
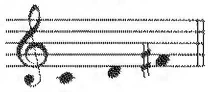
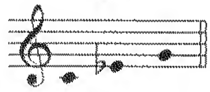
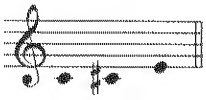
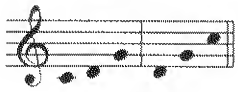
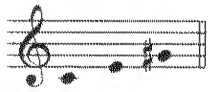
At the 'one o'clock' position on the larger clock-face is the 12-note triadic tonality of the first hour, at 'two o'clock' is that of the second hour, at 'three o'clock' that of the third hour, and so on through to the twelfth hour. (Note: the tenth hour shows the 12-note *tetradic* tonality, since in this hour no 12-note triadic tonality is possible.)

Naturally only one steering is shown, on each of the smaller clock-faces. Thus the first hour is shown steered by X^4 and the second hour by $VIII^4$ (in both cases, this is the only available steering that will produce all 12 notes without note-repetitions); the third hour is shown steered by $VIIm^4$ (though it can also be steered by V^4), and so on (see the IPF chart and p132 for the available steerings in the remaining hours).



Appendix V: Summary in Stave Notation

12 chromatic triads = 12 chromatic hours (fundamental interval patterns)

FIRST HOUR **I**  **1-1**	FOURTH HOUR **IV**  1-4	SEVENTH HOUR **VII**  2-3	TENTH HOUR **X**  **3-3**
SECOND HOUR **II**  1-2	FIFTH HOUR **V**  1-5	EIGHTH HOUR **VIII**  2-4	ELEVENTH HOUR **XI**  3-4
THIRD HOUR **III**  1-3	SIXTH HOUR **VI**  **2-2**	NINTH HOUR **IX**  2-5 = **5-5**	TWELFTH HOUR **XII**  **4-4** (twelfth-hour subscale)

arabics = intervals in semitones S = small/short interval L = large/long interval

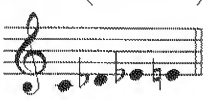
S-L = *minor triad* L-S = *major triad* (inversion) EV^2 (2 equal values)

asymmetrical triad symmetrical triad

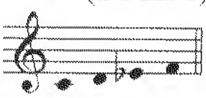
LARGER HOUR-GROUPS

symmetrical tetrad(ST): SLS (minor) LSL (major) EV^3 (3 equal values)

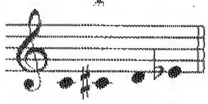
E.g.: IIm^4 (STIIm) IIM^4 (STIIM) I^4



121[c]



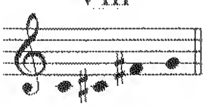
212[c]

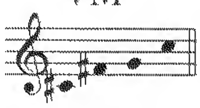


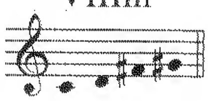
111[c]

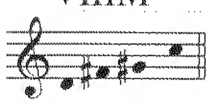
NB: exceptional STs: fifth hour and eighth hour

Vm^4 = VM^4 $VIIIm^4$ = $VIIIM^4$







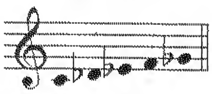


151[c] = **515**[c#] **242**[c] = **424**[d]

(fifth-hour subscale) (eighth-hour subscale)

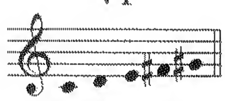
symmetrical pentad(SP): SLLS (minor) LSSL (major) EV^4 (4 equal values)

E.g.: SP IIm SP IIM VI^5



1221[c]

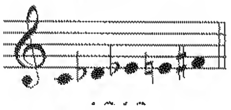
2112[c]

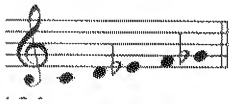


2222[c]

oedipus pentad: SLSL (minor) LSLS (major)

E.g.: IIm^5 IIM^5 (inversion of IIm^5)

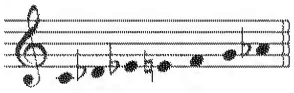




LARGER HOUR-GROUPS (continued)

gemini: SLS-SLS (minor) LSL-LSL (major)

E.g.: second-hour minor



121-121

second-hour major

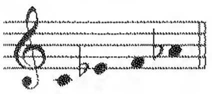
212-212
(diatonic scale, aeolian mode)

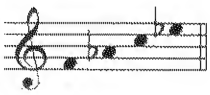
	minor	major
greater gemini:	SLLS-SLLS	LSSL-LSSL
gemini triplet:	SLS-SLS-SLS	LSL-LSL-LSL
oedipus twin:	SLSL-LSLS	LSLS-SLSL

MULTIPLE-NATURE HOUR-GROUPS

E.g.: double-nature STs

VIIM^4 = XIm^4



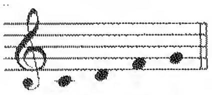


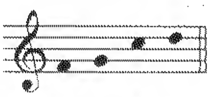
323[c] = **343**[f]

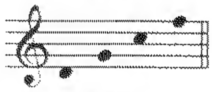
IVM^4 = XIM^4

414[c] = **434**[f]

VIIm^4 = IXm^4 = IX^4



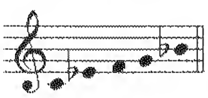


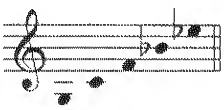


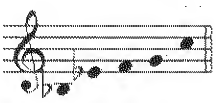
232[c] = **252**[f] = **555**[d]

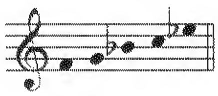
E.g.: pentatonic scale

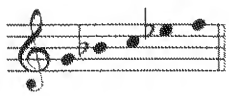
SP VIIM = IX^5 = SP IXM = VIIm^5 = VIIM^5 (oedipus pentads)







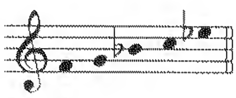




3223[c] = **5555**[g] = **5225**[b^b] = **2323[f] = **3232[g]

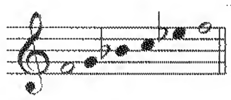
'IMPOSSIBLE' SYMMETRIES: shown by double asterisk **

E.g.:



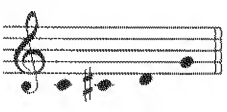
**2323
asymmetrical pentad

=



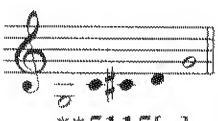
****23232**
'impossible' symmetrical hexad

E.g.:



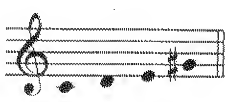
**115[c]
asymmetrical tetrad

=



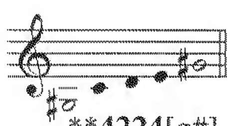
****5115**[g]
'impossible' symmetrical pentad
NB: also = ****1551**[c#]

and



**224[c]
asymmetrical tetrad

=



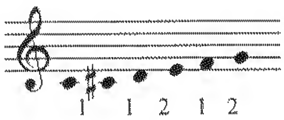
****4224**[g#]
'impossible' symmetrical pentad
NB: also = ****2442**[d]

STEERING EXAMPLES = COMPOUND TRANSPOSITION

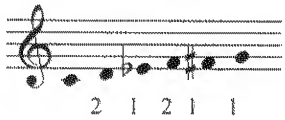

10 6/1[e]
Vm4
1/6[e]
5th-hour subscale
11 Vm4/3[d]
IIm8
3/Vm4[d]
2nd-hour subscale
12 VM4/XII[c#]
I12
XII/VM4[c#]
1st-hour scale
13 IX/X4[c]
I12
X4/IX[c]
14 IX(mMmM)/IIM4[c]
m M m M
I12
15 2/1[f#]
I4
1/2[f#]
16 IVm/X4[c#]
I12
X4/IVm[c#]
1st-hour scale
17 IVM/X4[d]
I12
X4/IVM[d]
18 VIIM4/6[c#]
IIM8
6/VIIM4[c#]
2nd-hour subscale
19 XIM/IX[g]
212-212[a]
IX/XIM[g]
2nd-hour major gemini

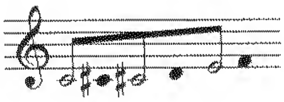
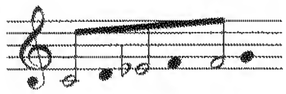
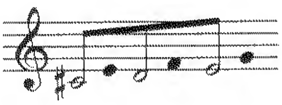
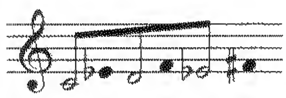
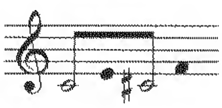
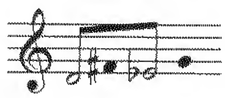
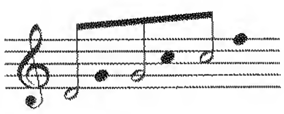
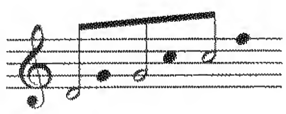
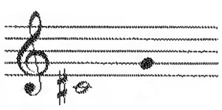
INTERVAL ARRAY

E.g. the hexad 6.11(Z40) 11212 (see *Chromatic Map I*, p49)



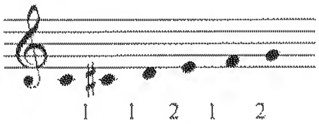
Interval Array					
(1	2	3	4	5	6)
3	3	3	2	3	1



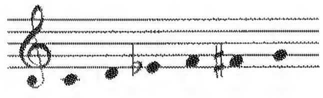
(see Array Steerings Chart, *Chromatic Map II*, p85)			INVERSION (not shown in AS Chart)	
3 x semitones		**1/** IIIm	**1/**IIIM[2]	
3 x wholetones		**2/**VIIm	**2/**VIIM	
3 x minor 3rds		**3/**IIm[1]	**3/**IIM	
2 x major 3rds		**4/1**	**4/1**[2]	
3 x perfect 4ths		**5/**IX[2]	**5/**IX[2]	
1 x tritone		**6**[1]	**6**[0]	

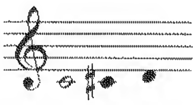
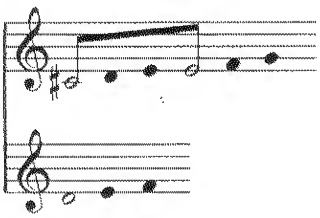
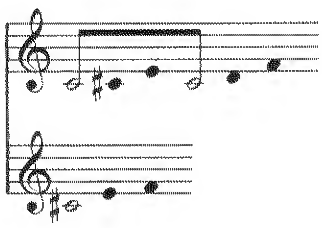
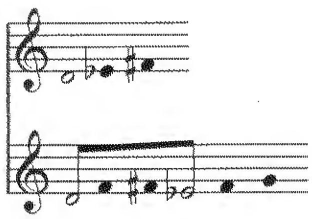
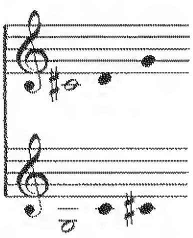
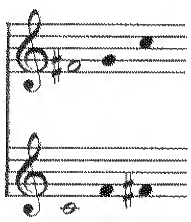
TRIAD ARRAY

E.g. the hexad 6.11(Z40) 11212 (see *Chromatic Map I*, p49)

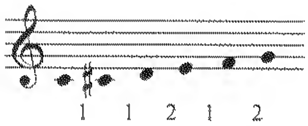


Triad Array					
(I	II	III	IV	V	VI)
1	3	3	2	2	1
3	1	2	1	1	0
(VII	VIII	IX	X	XI	XII)



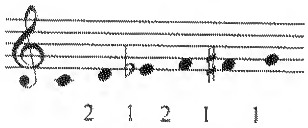
(see Array Steerings Chart, *Chromatic Map II*, p85)			INVERSION (not shown in AS Chart)	
1 x Triad I		I[0]	I[5]	
3 x Triad II		IIm/**3**[1] IIM[2]	IIm[2] IIM/**3**	
3 x Triad III		IIIm/**1** IIIM[1]	IIIm[2] IIIM/**1**[2]	
2 x Triad IV		IVm[0] IVM[0]	IVm[2] IVM[2]	
2 x Triad V		Vm[1] VM[7]	Vm[6] VM[0]	

TRIAD ARRAY (cont.)



Triad Array

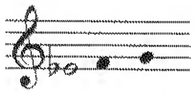
(I	II	III	IV	V	VI)
1	3	3	2	2	1
3	1	2	1	1	0
(VII	VIII	IX	X	XI	XII)



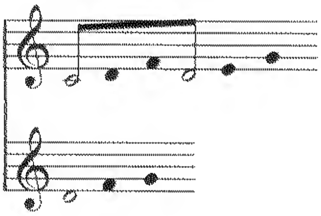
1 x Triad VI

VI[0]

VI[3]



3 x Triad VII

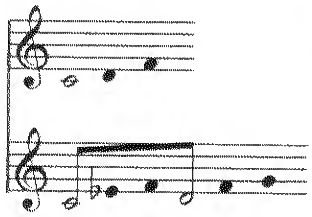


VIIm/**2**

VIIM[2]

VIIm[0]

VIIM/**2**



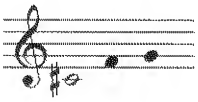
1 x Triad VIII

VIIIm -----

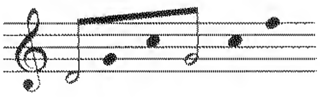
VIIIM[1]

VIIIm[0]

VIIIM -----

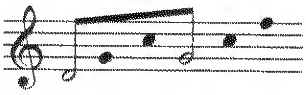


2 x Triad IX

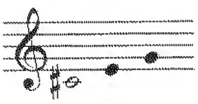


IX/**5**[2]

IX/**5**[2]

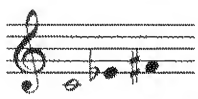


1 x Triad X



X[1]

X[0]

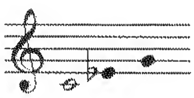


1 x Triad XI

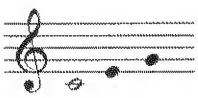
XIm -----

XIM[0]

XIm[0]



XIM -----



0 x Triad XII

XII -----

XII -----

'MINI TONE-CLOCK-TYPE' HARMONIC FIELDS

formed within Messiaen's modes

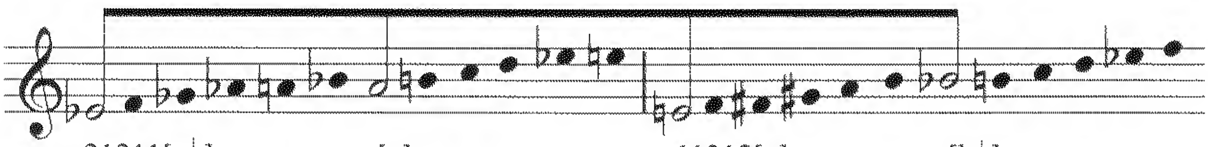
(see *Chromatic Map II*, pp74-75)

FIELD A

Mode 7: 1111-2-1111

Mode 7 (3°)

21211 (+ inversion) steered by Vm^4[e♭ or a]

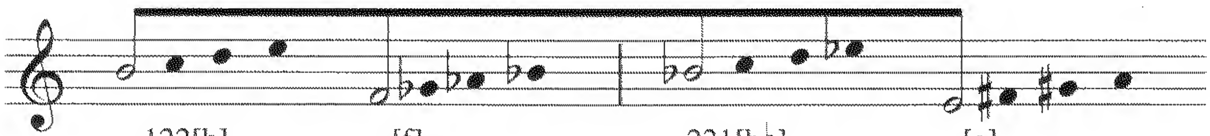


The chosen original group 21211[a] is found in Mode 7, transposition 3, where it is steered by a tritone, and its inversion 11212 is also found, steered by another tritone (all of this is *axiomatic* for *any* subset of Mode 7). The above symmetrical harmonic field, containing only the 10 notes of Mode 7, transposition 3, can therefore be created.

Mode 6: **1221-1221 *Modal Complements*

(1°) (5°)

122 (+ inversion) steered by Vm^4[e or b♭]

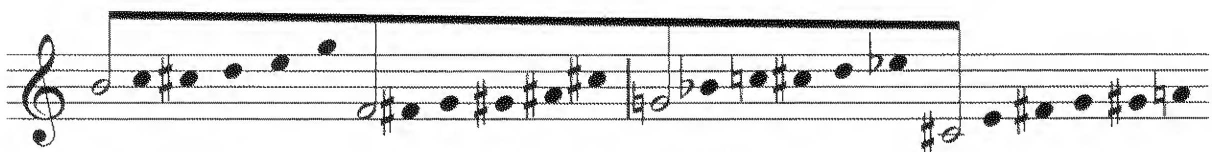


The modal complements (i.e., the *remaining* notes of Mode 7) of each group, in turn, of the original field above form this modal-complementary field in Mode 6, using transpositions 1 and 5.

Mode 7 *Chromatic Complements*

(5°) (1°)

11123 (+ inversion) steered by $VIIIm^4$[f or b]



The *chromatic* complements (i.e., the remaining notes of the *twelve* available) of each group, in turn, of the original field above form this chromatic-complementary field which is also in Mode 7, using transpositions 5 and 1.

'MINI TONE-CLOCK-TYPE' HARMONIC FIELDS (cont.)

FIELD B

Mode 7: 1111-2-1111

Mode 7 Transpositions 4° 1° 3° 5° 2° 6°

Modal Complements

3223/3223 [f#] (= **5555** [c#])

Mode 7 Transpositions 4° 1° 3° 5° 2° 6°

3223/22122 [a] (= **55555** [c#])

The chosen group **3223**[f#], which is found in Mode 7, is here steered by itself (white notes). The black notes show the modal complement for the group beneath. Note that **3223** is complementary to *itself*, in Mode 7. Note also that whenever a symmetrical group is steered by itself (or whenever an asymmetrical group is steered by its inversion), one note always remains constant, being present in each group transposition within the given steering configuration, and taking the function, in turn, of every note in the group (e.g., highest note, 2nd highest note, and so on). In the above example, this constant note is [e], which can be regarded as a 'structural' tonic, since a tonic effect is created structurally, by applying a logical chromatic technique. The equivalent note in the modal complements above is [b♭].

When the last transposition, transposition 6, is added, a second steering possibility arises, **3223** steered by **22122**. The two steering-groups, **3223** and **22122**, are related in that they both have alternative ninth-hour forms, through their multiple hour-natures.

Appendix VI: Steering-Partners & Anchor-Forms

Steering-Partners

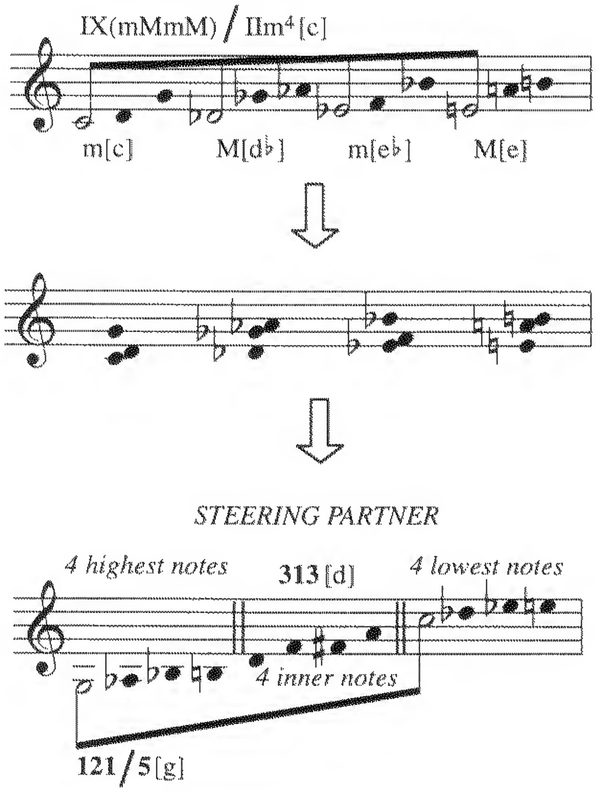
On page 71, reverse steerings are discussed in connection with what I call the 'steering-partner' relationship. Further reverse steerings can also be seen in stave notation in Appendix V, pp116-7.

The steering-partner relationship takes three main forms.

The most global form concerns simply a steering relationship between two *hours*, regardless of whatever particular *groups* in those hours may be involved. Thus if, say, the first hour steers the seventh hour in a given steering configuration, then in the most generalised steering-partner for this configuration, very simply, the seventh hour will steer the first hour. A tetrad may steer pentads in the original, conceivably, and a heptad may steer triads in the steering-partner—any differences in the actual group sizes are immaterial. If the steering relationship between the *two hours themselves* is reversed, then this is sufficient, in the most general terms, for a steering-partner relationship already to exist.

A more specific form of steering-partner is the reverse steering, in which both the hours *and* the actual group sizes are reversed in the steering-partner configuration. The reverse-steering examples mentioned on page 71 and those illustrated on pages 116-7 are all steering-partner relationships of this kind. Such steering-partnerships always produce exactly the same notes (in total) as those produced by the original configurations themselves (which will not necessarily be the case with the more global steering-partners discussed above).

In the most intricate form of steering-partner relationship, the structure and hours produced in the steering-partner are somewhat *different* from those that exist in the original steering configuration, although the actual notes involved are the same. The following is an example of this kind of steering-partner:



The unlisted reverse steerings on pages 116-7 are all similarly idiosyncratic.

Anchor-Forms

The steering-partner in the example above is a new type of symmetrical hour-structure that I refer to as an 'anchor-form'.

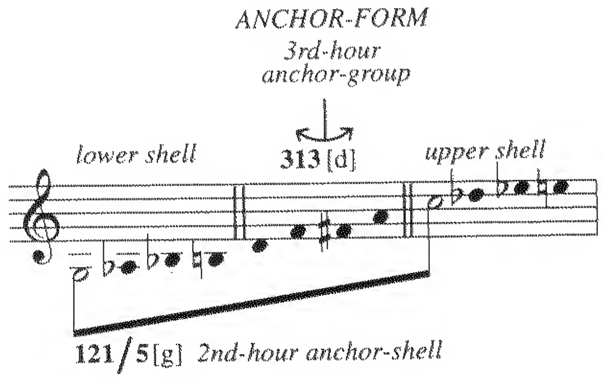
An 'anchor-form' is a (spatially) symmetrical pitch structure with a central 'anchor-group' (which may be a single note, an interval, triad or larger group) which has a *single* chromatic colour or tonality (or hour), surrounded by one or more 'shells', each of which *also* has a single chromatic colour or tonality, normally *different* from that of the actual anchor-group (just as any multiple shells will likewise have diffent colours from one another and from the anchor-group).

The term 'chromatic colour' may be understood more-or-less in the same way that a painter would understand the idea of 'colour' and 'tonality'. In painting, the colour spectrum is divided so as to provide a range of main or primary colours which are then mixed by the painter to form further colours. Similarly, in today's equal-tempered music, the sound spectrum is divided so as to produce the twelve chromatic notes, each of which has its own particular 'colour' (enhanced compositionally, of course, by its actual musical timbre or tone-colour, as well as its register in relation to the general tessitura, or most comfortable range, of that particular instrument or voice). A single note has no actual 'chromatic tonality' as such (in the sense that Schat uses this term) *on its own*, but only in relation to *other* notes. But if this single note is used as a 'referential' tonic—i.e., as a particular pitch to which the ear can keep referring—then it can be thought of as having 'tonality' (or 'tonicality') in the old 'primal' (as opposed to classical or functional) sense. These are the implications, at any rate, when a single note is the centre, or anchor-note, of an anchor-form.

Beyond and subsuming the twelve notes, there are the six intervals, which have their own even more marked 'colour' or 'ambience', or 'climate'. The intervallic colours then extend into the twelve chromatic triads, with a further colour emphasis or differentiation created either by the repetition of two intervals that are the *same* (in the symmetrical triads) or by the mixture of two *different* intervals (in the asymmetrical triads). Thus the minor 3rd on its own, for example, can be considered to be a sort of primal tenth hour, and so on. We could keep on extending this 'colour-mixing' even further, to account for the symmetrical and asymmetrical tetrads, and indeed eventually for all of the possible chromatic groups. But the main point to note in connection with the concept of 'anchor-forms' is that the central anchor-group has a different 'colour' or 'hour' from that of its shell (and that any multiple shells will likewise be coloured differently from each other).

The anchor-group (indicated below by the little anchor-sign) is naturally always in an exactly inverse relation to the two halves of the shell (which I call the 'lower shell' and the 'upper shell', respectively)—thus it acts as a 'mediator'. But more importantly for the composer, the musical realisation of an anchor-form will suggest most obviously that the anchor-group might receive a different musical treatment from that of the shell (or shells).

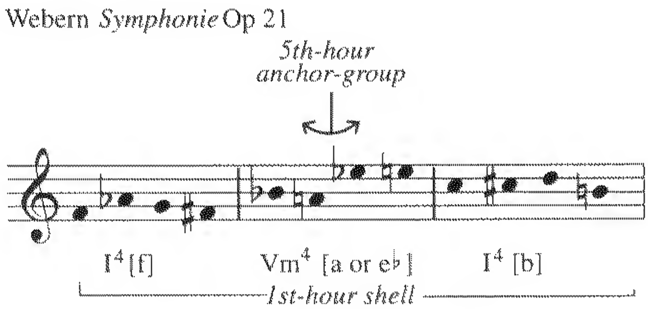
The steering-partner shown in the last example above can be interpreted as an anchor-form, as follows:



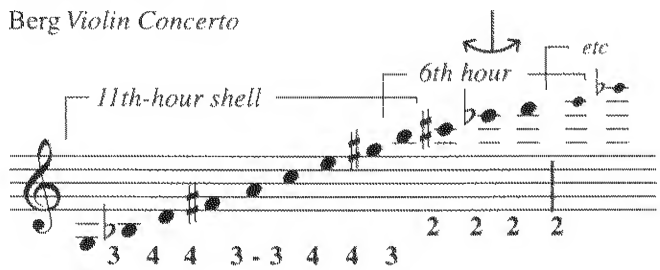
Needless to say, the upper and lower shells in this example could perfectly well exchange places, in which case **121** (or IIm^4) would then be steered by a perfect 5th (**7**) instead of a perfect 4th (**5**). (This move, incidentally, is a handy form of compound permutation, like the half-turn of an oven switch, that I call 'intrinsic spin'. It can be applied, either horizontally or vertically, to two notes [or elements] or to two *groups* of notes [or elements], e.g.: or Berg, Webern and Stravinsky, at least, have also used it.)

The foregoing example is a 12-note anchor-form, although an anchor-form as such may also contain fewer or more than twelve notes (i.e., with or without note-repetitions). Virtually all of the possible 12-note (tone-clock) triadic tonalities that can be formed by the asymmetrical triads (see p132) have tetradic steering-partners that are 12-note anchor-forms similar to the above.

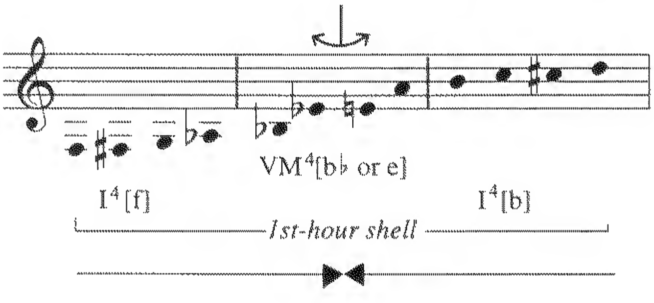
'Anchor-forms' are not purely my own invention. The following examples show two 12-note series from a couple of well-known early 20th-century works. Both of these series can (and, I would argue, should) be regarded as 12-note anchor-forms. The first is the series of Webern's *Symphonie*, Op. 21, as it appears in the second movement (written first, I seem to recall). The anchor-form interpretation is particularly valid, since Webern certainly treats his fifth-hour anchor-group in a notably different way from the first-hour shell (giving it conspicuously to the harp):



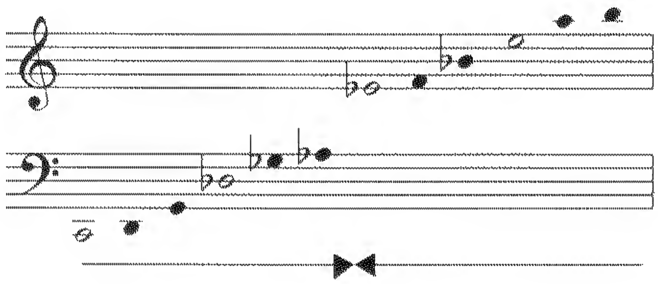
The second example is the series from Berg's *Violin Concerto,* a series which one might justifiably suspect has already well-embedded itself in the memory of most of us—and only, I would hold, because of the essentially simple and memorable intervallic relationships that constitute the eleventh and sixth hours respectively:



If we were to continue this same series cyclically, as suggested by the last two small notes in the example above, then we would end up with a symmetrical chord 'rolling' through the registers, a 'rolling' anchor-form. Every anchor-form can *always* be arranged as a symmetrical chord. Here is Webern's Op. 21 series, no longer expressed as an ordered 12-note group, but now laid out (along tone-clock lines) as a symmetrical chord. (Webern himself organised the first movement spatially/vertically according to several *other* [essentially ninth-hour] symmetrical chords). This is the fundamental anchor-form of which Webern's series is but one (of many possible) ordered versions:



Likewise, every simple 12-note tone-clock tonality can also always be rearranged spatially as a symmetrical chord, as in the example below, for instance (a rearrangement of the first example in this appendix), which in its present ninth-hour grouping is *not* an anchor-form :



The above example could easily enough, however, be regrouped (or treated musically) as an anchor-form, if for instance the central four notes were taken together as a second-hour anchor-group (**212**[e♭]), and the two outer tetrads were treated as a **115 anchor-shell (so as to fulfil the anchor-form condition of a single [**115–**511] colour for the shell, different from the colour/treatment of the anchor-group).

An anchor-form is a harmonic field—in principle, its components (anchor-group [or anchor-note], upper and lower shells [single or multiple]) may come in any order, and the notes within any component group may also come in any order. As with the regular tone-clock tonalities, what holds a group together is its chromatic *colour*—only, in an anchor-form, the colours (and often the group densities also) are different, not the same. The symmetry may be present only in the deep structure, with the notes in the actual music being treated perfectly freely as regards register. Alternatively, the symmetry may also be manifested in the music, by arranging the anchor-form as a fixed-pitch symmetrical chord—which can then itself be subjected to various intrinsic spins, steered at a deeper level, and so on. Regrouping (so as to change the implied structure, and thus the colours—as suggested above, for example) is another rich source of variation.

Note: an anchor-group is usually smaller than its shell. If the same size as the shell, then it is a moot point which is the anchor-group, and which the shell (though this need not be any great problem, since the notes can always be rearranged with the shell instead of the anchor-group in the centre, due to the 2nd law of chromatic symmetry). If the shell is *smaller* than the anchor-group then the two have essentially exchanged functions—indeed, an evolving interchange or dynamic along these lines may be consciously created.

Appendix VII: Constructing Symmetrical Harmonic Fields & Anchor-Forms

Any axis-symmetrical group (i.e., with an even number of notes) can be used to steer any symmetrical group (or any asymmetrical group and its inversion) in such a way as to form a symmetrical harmonic field. *Note*: if the group steered is asymmetrical, the original and inversion must be *reciprocally placed* so as to *mirror each other* in the two symmetrical halves of the field (see examples overleaf, in which the eleventh-hour triad is used, purely because it is the most familiar chromatic group). If the resulting harmonic field uses fewer than 12 notes, then the remaining, unused chromatic notes will always form an anchor-group (1st law of chromatic symmetry), thus transforming the whole into an anchor-form, with the harmonic field as the shell.

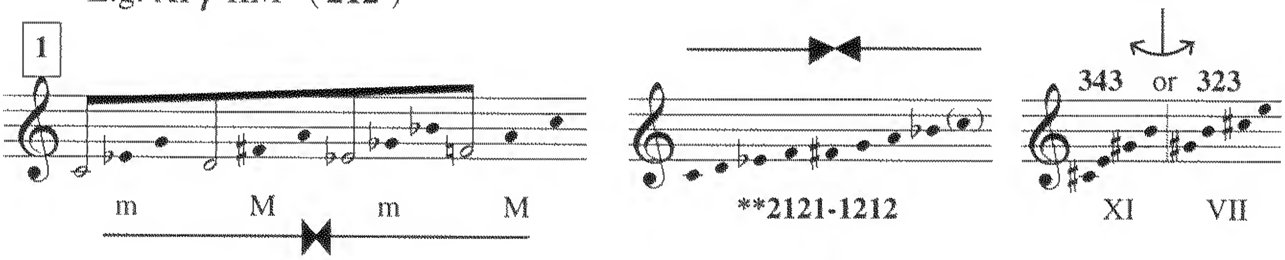
Any point-symmetrical group (i.e., with an odd number of notes) will always form a symmetrical harmonic field if it steers a symmetrical group. If the group steered is asymmetrical, there are two possible treatments. You can either treat the central note (or any central symmetrical group) of the point-symmetry simply as an anchor-note (or anchor-group), i.e., not used to steer anything, and for the rest proceed as above. In this case, any remaining unused chromatic notes (of the twelve) will form an *additional* anchor-group, and you will have an 'anchor-within-an-anchor'. Or, alternatively, you can use the central note of the point-symmetry to steer (in succession) both the original *and* the inversion of the asymmetrical group being steered, and for the rest proceed as above. Any remaining unused chromatic notes will once again form an anchor-group.

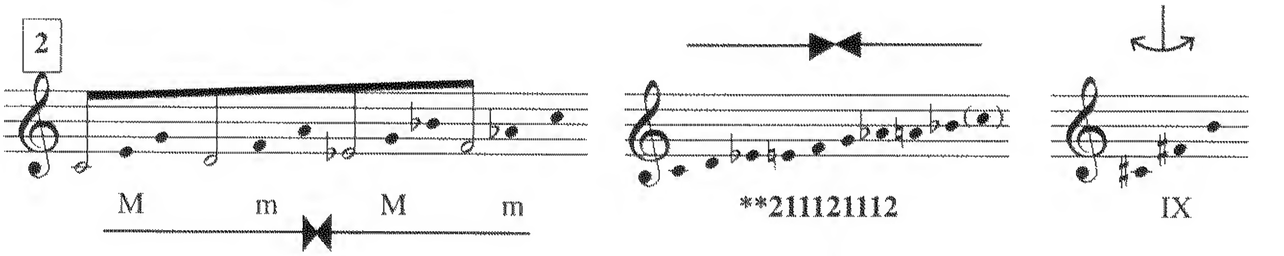
Through the 1st law of chromatic symmetry, the aforesaid principles will naturally apply equally to any 'mini tone-clock-type' harmonic fields created within Messiaen's modes (as discussed on pp74-75 and illustrated on pp121-22). E.g., for the first (as well as the second) field on p121, the anchor-group is the remaining unused tritone g-c#, chromatic complement of Mode 7, transposition 3. At first sight, the third field on p121 has no *overall* chromatic-complementary anchor-group, since all 12 notes are used, but each *half* on its own has an anchor-group (the tritone complements of Mode 7, transpositions 5° and 1°, respectively). Similarly, in Field B (p122), which also uses all 12 notes, each of the six Mode 7 transpositions taken separately has an anchor-group (its respective tritone complement). Thus a *larger* ('tritone-coloured') anchor-group can 'accumulate'—itself eventually also containing all 12 notes—as one progresses through the various transpositions, which themselves will form the larger ('**3223**-coloured') shell.

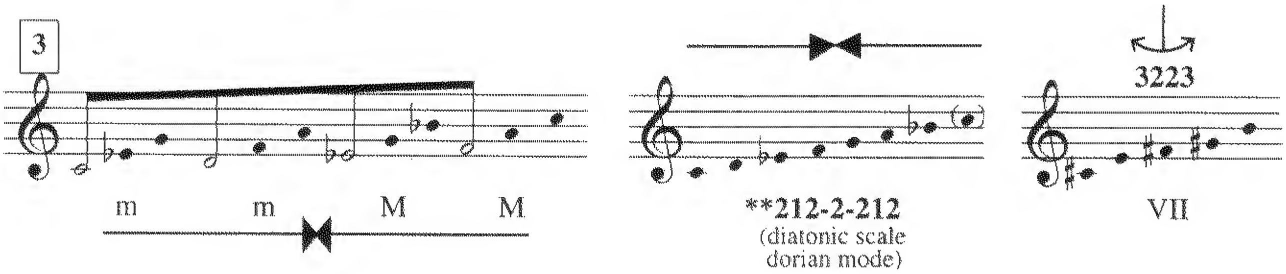
The same principles will also apply wherever the 'binding' process is involved (see Appendix VIII)—i.e., the chromatic complement of a binding-group can always serve as an anchor-group, with the binding-group forming the shell.

Notes used in the field *Remaining chromatic notes*

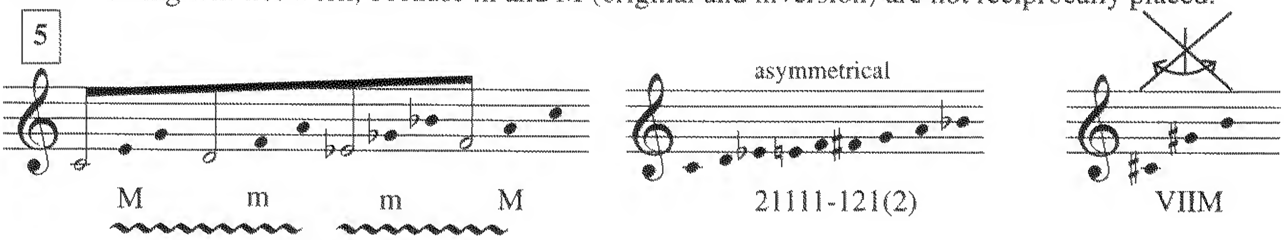
E.g. XI / IIM^4 (**212**)

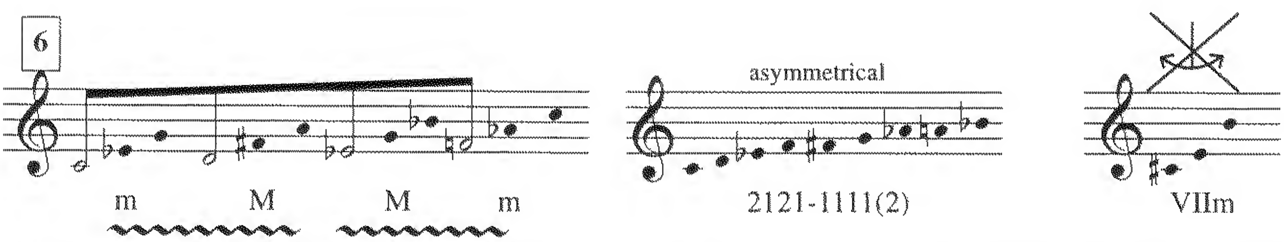






The following will not work, because m and M (original and inversion) are not reciprocally placed.



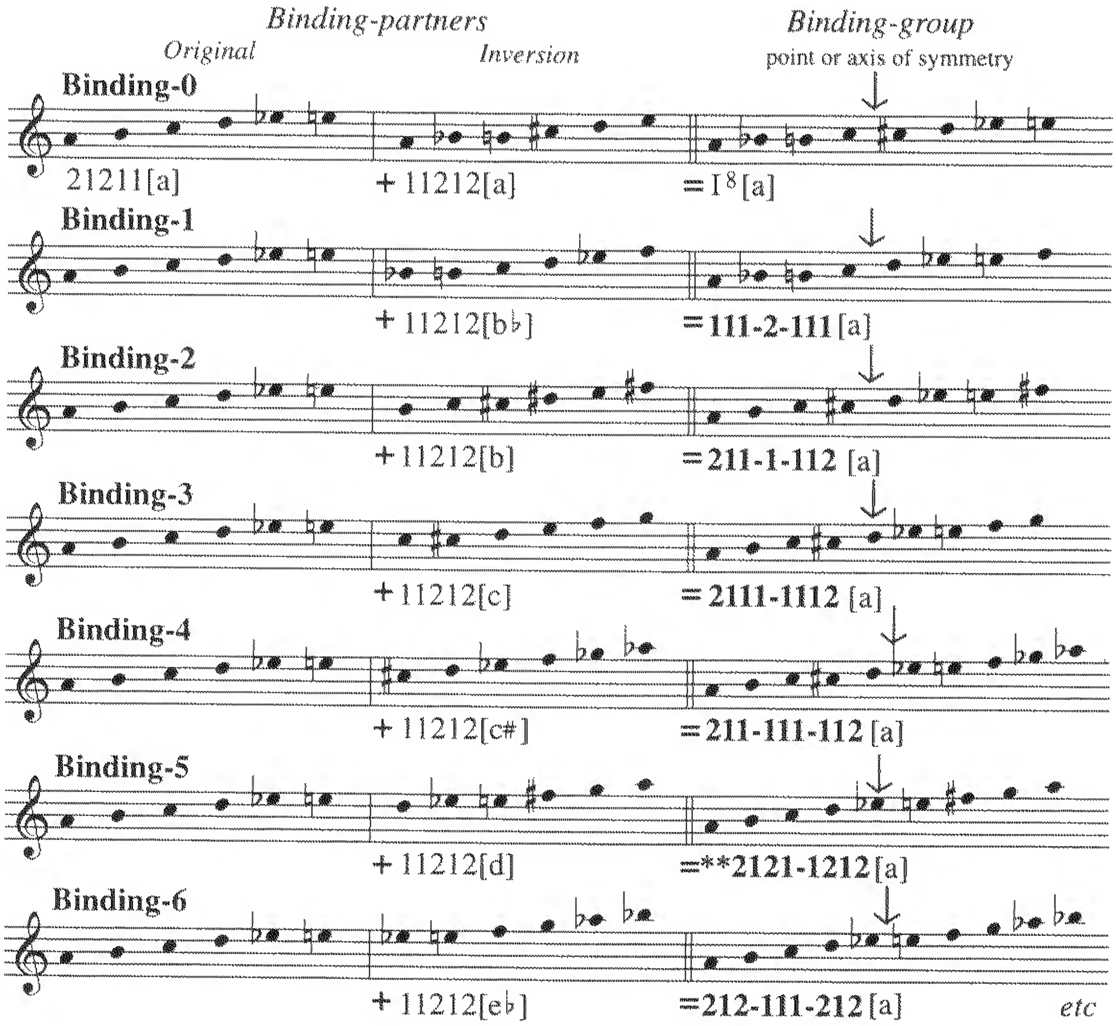


NB: Field 6 *is*, however, the inversion of Field 5, so the two *together* can form a larger symmetrical field, with a combined anchor-group **323**[g#], or **343**[c#].

Appendix VIII: Binding

In binding, an asymmetrical group is superimposed on its inversion (or a symmetrical group is superimposed on itself), so as to form a new symmetrical group. In 'zero binding' (Binding-0), original and inversion are superimposed in such a way that their respective outer notes are the same. In the remaining bindings, the distance between the respective outer notes is progressively enlarged. Thus a whole 'binding network' can be created. E.g.:

21211 BINDING NETWORK



In terms of the total notes produced (i.e., not counting note-repetitions), 21211 Binding-7 is, of course, the same as 11212 Binding-5, 21211 Binding-8 is the same as 11212 Binding-4, and so on. (Likewise, 11212 Binding-7 is the same as 21211 Binding-5, etc.) However, the actual disposition of the notes, in terms of range and octave-repetitions, is different in each case, so it can be worthwhile keeping these distinctions.

Since the binding-groups formed by the various bindings are different sizes and contain varying numbers of pitch classes, their respective chromatic complements, which can serve as anchor-groups, are different sizes as well. Thus, moving through the network, both the anchor-group component and the shell (or binding-group) component will keep on changing their sizes—a further property of interest to the composer.

On the opposite page, the simplest binding-groups in the chromatic system—the symmetrical tetrads formed by zero-bindings of the asymmetrical triads—are shown. It is useful to get to know these simplest binding-relationships, since the various triad binding-partners concerned crop up frequently in permutations of these STs. The triad binding-partners are also the only available triad subsets in an hour that is *different* from the hour of the ST itself—or a 'partner' hour. As shown opposite, an ST has at most *one* partner hour, and some STs have *no* partner hour, namely V^4, $VIII^4$ and X^4 (the number of partner hours contained in a chromatic group is indicated in its triad array).

Any symmetrical group larger than a tetrad (including the impossible symmetries) can always be treated as a binding-group, and broken down into various binding-partner components. Often, indeed, numerous possible binding-partners can be extracted from a given symmetrical group (due to the 3rd law of chromatic symmetry).

Thus, a binding-group can be constructed from one particular chromatic group, and then new binding-partners can be found within it.

Also, any binding-group or symmetrical group larger than a tetrad can, of course, always be transformed *within itself* into an anchor-form—by extracting from it an anchor-group and shell, in such a way that all the notes of the original group are used (with or without note-repetitions). The chromatic complement of the original will then form a second anchor-group, if necessary, and once again you will have an 'anchor-within-an-anchor'.

It will be seen that 'binding' and 'steering' are more-or-less just slightly different versions of the same thing. In a binding-3, for example, we could say that the given group (original followed by inversion) is steered by a minor 3rd, and then the two resulting transpositions are collapsed together into a single group. It will be seen, too, that 'mini tone-clock' fields and the other symmetrical harmonic fields discussed (pp127-8) are simply an extension of the binding principle, without any collapsing of the steered groups (although collapsing could certainly be introduced).

The following inter-relationships also exist, formed through simple binding networks, amongst Messiaen's modes and their chromatic complements:

Tritone Binding Network

	Binding-Group formed	Chromatic Complement
Zero-binding	Tritone	Mode 7
Binding-1	Vm^4	Mode 4
Binding-2	$VIIIm^4$	Mode 6
Binding-3	X^4	Mode 2

Vm^4 Binding Network

Zero-binding	Vm^4	Mode 4
Binding-1	Mode 5 (**11411**)	Mode 5 (**11411**)
Binding-2	Mode 4	V^4
Binding-3	IIm^8 (Mode 2)	X^4

$VIIIm^4$ Binding Network

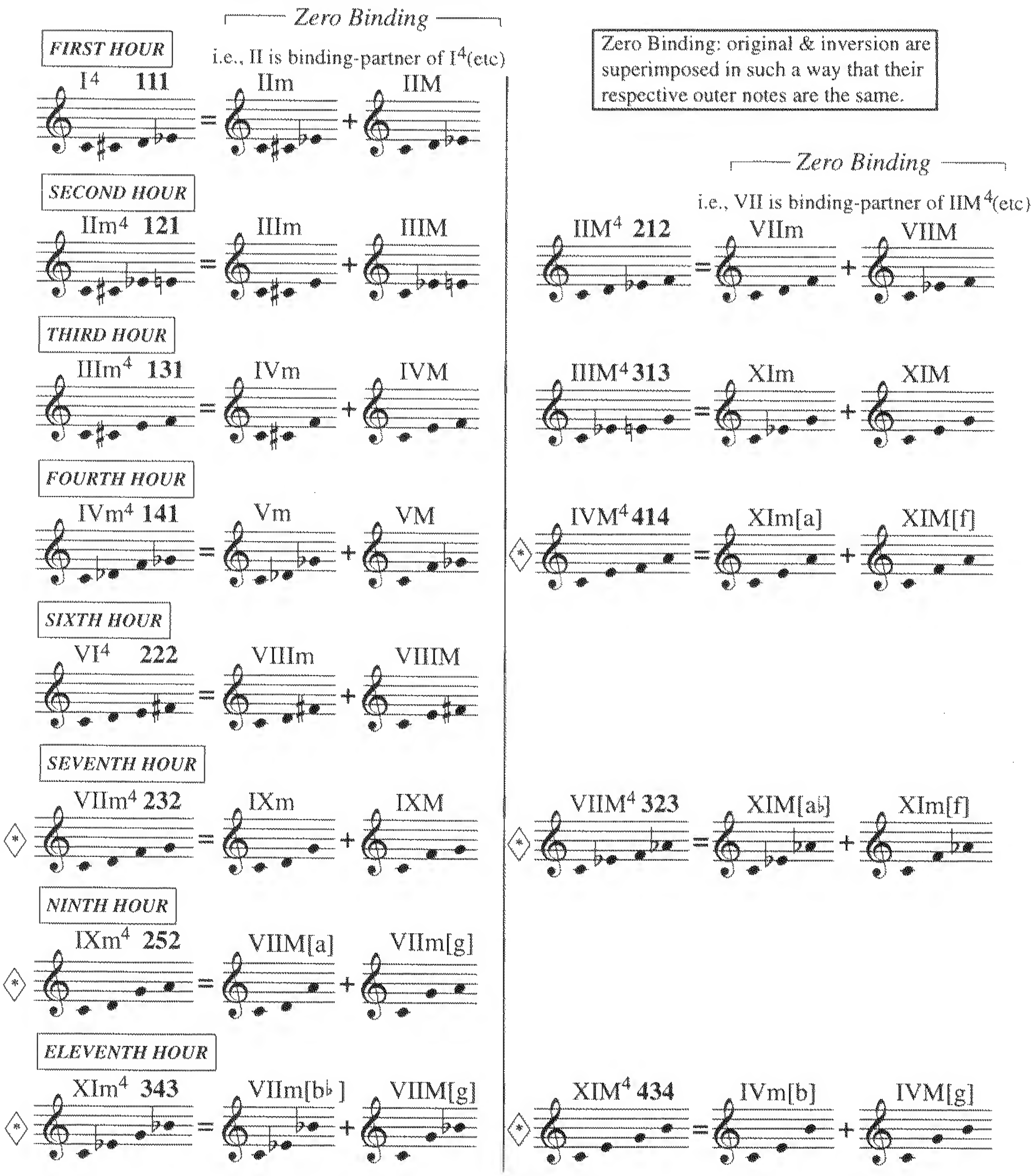
Zero-binding	$VIIIm^4$	Mode 6
Binding-1	Mode 4	V^4
Binding-2	Mode 1 (VI^6)	Mode 1 (VI^6)
Binding-3	IIM^8 (Mode 2)	X^4

X^4 Binding Network

Zero-binding	X^4	Mode 2
Binding-1	IIm^8 (Mode 2)	X^4
Binding-2	IIM^8 (Mode 2)	X^4

SYMMETRICAL TETRAD BINDING-PARTNERS

It is handy to become familiar with these simplest binding relationships, since they help one, in analysis, to recognise the presence of a symmetrical tetrad, and are an easy way to account for the 'foreign' (in hourly terms) triads that emerge in various permutations & subsets of most STs.



(*) Starred STs are also double-nature hour-groups:

i.e. **414** (4th hour) = **434** (11th hour)
232 (7th hour) = **252** (9th hour) = **555** (9th hour)
323 (7th hour) = **343** (11th hour)

It will be seen above that the hour of their binding-partner is the same as the hour of their double-nature partner.
Note: symmetrical tetrads can have at most *one* partner hour, and some have *no* partner hour (the 5th-hour, 8th-hour and 10th-hour STs)

Appendix IX:
12-Note Triadic Tone-Clock Tonalities (in Stave Notation)

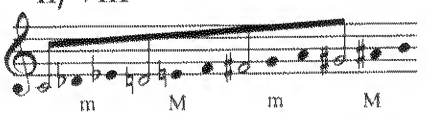
NB: The three notes *within* each triad may come in any order, and the four triads in each field may also come in any order. Transpositions refer to the number of possible transpositions before the same triad pitch-class content is repeated (regardless of note-order or triad-order, and regardless of which notes happen to be the steering-notes).

FIRST HOUR

I/X^4 3 transpositions

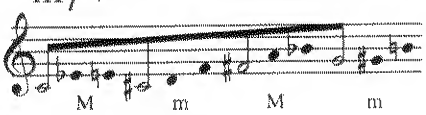
SECOND HOUR

II/VIII^4 6 transpositions

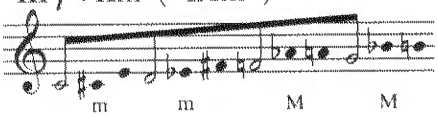


THIRD HOUR

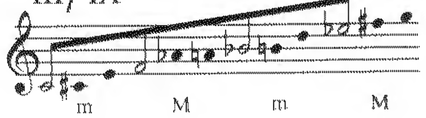
III/V^4 6 transpositions



III/VIIm^4(=IXm^4) 12 transpositions

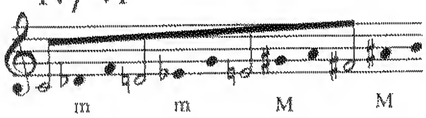


III/IX^4 12 transpositions

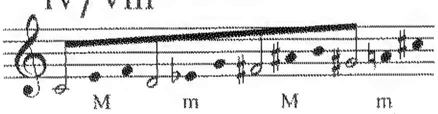


FOURTH HOUR

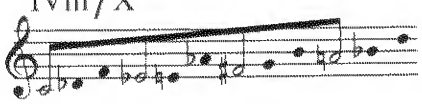
IV/VI^4 12 transpositions



IV/VIII^4 6 transpositions



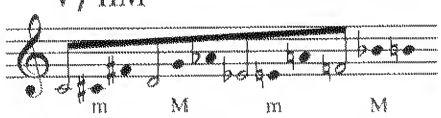
IVm/X^4 3 transpositions



IVM/X^4 3 transpositions

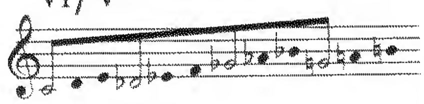
FIFTH HOUR

V/IIM^4 12 transpositions

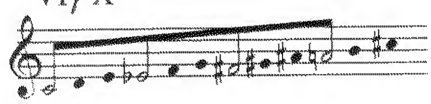


SIXTH HOUR

VI/V^4 6 transpositions

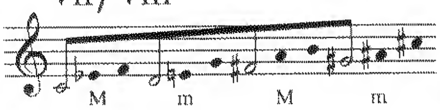


VI/X^4 3 transpositions



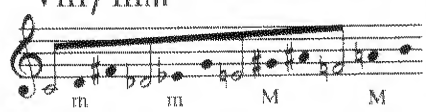
SEVENTH HOUR

VII/VIII^4 6 transpositions

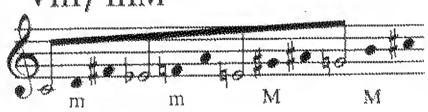


EIGHTH HOUR

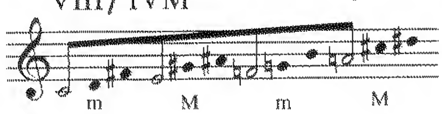
VIII/IIIm^4 12 transpositions



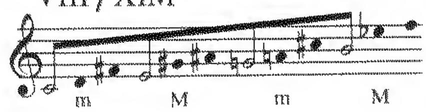
VIII/IIIM^4 12 transpositions



VIII/IVM^4 12 transpositions

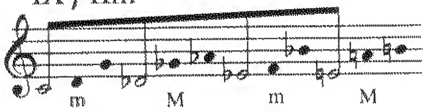


VIII/XIM^4 12 transpositions

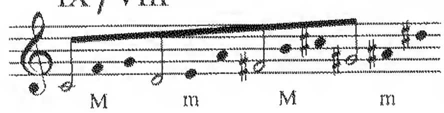


NINTH HOUR

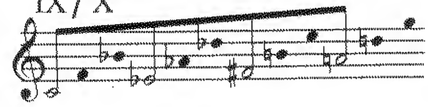
IX/IIm^4 3 transpositions



IX/VIII^4 3 transpositions



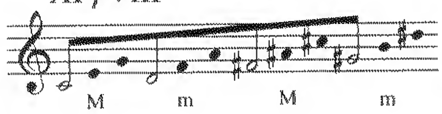
also
IXm/X^4 & IXM/X^4
IX/X^4 3 transpositions



ELEVENTH HOUR

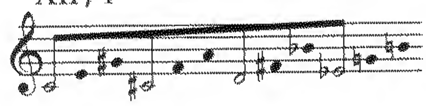
XI/VI^4 12 transpositions

XI/VIII^4 12 transpositions

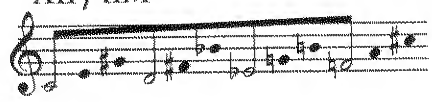


TWELFTH HOUR

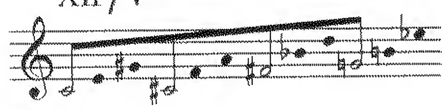
XII/I^4 0 transpositions



XII/IIM^4 0 transpositions



XII/V^4 0 transpositions

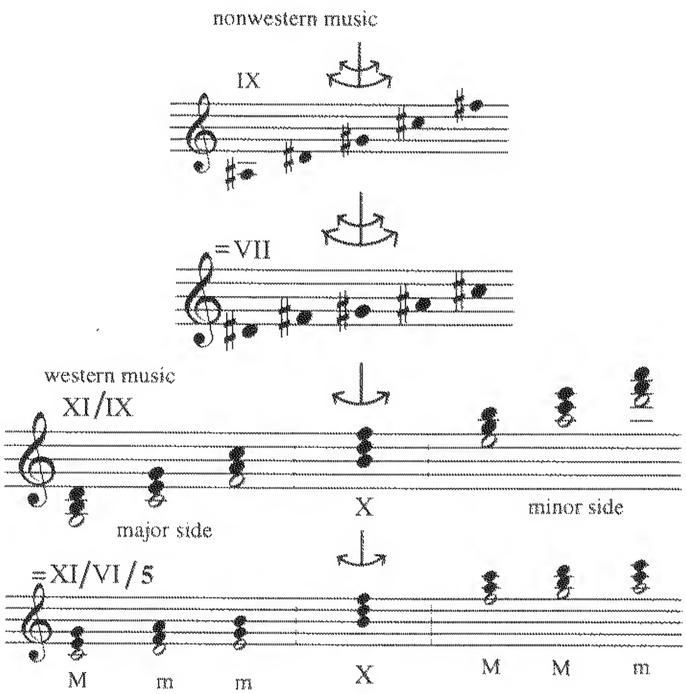


also IXm^4
XII/IX^4(= VIIm^4) 0 transpositions

XII/X^4 0 transpositions

Interestingly enough, I have never yet come across a 12-note series (including many invented at random) that could not be identified at bottom either as an ordered version of one of the regular 12-note tonalities (triadic, see opposite, or tetradic, see pp41, 43) or as a 12-note tetradic anchor-form, or else as only an intrinsic spin away from one of these[1]. Clearly, it will have some further bearing on compositional possibilities, and on our general comprehension of the chromatic system, if every 12-note series should prove to be as close as this to one of these 'natural' chromatic deep structures[2].

A further observation: if we are prepared to accept loosely that the bulk of traditional non-western music is based on the pentatonic scale (or portions of it) and the bulk of traditional western music is similarly based on the diatonic scale—leaving aside the question of specific and non-equal-tempered tunings for the moment—then it is possible to see 'world music' in a nutshell, as a 12-note anchor-within-an-anchor-form:



The western diatonic system in itself can be seen as a 7-note anchor-form, with triad X as the anchor-group. The 'non-western' pentatonic scale, expressed as the chromatic complement, and symmetrical here around g#, is also symmetrical around d (2nd law of chromatic symmetry), like the triad X anchor-group beneath it (and vice versa, of course). Thus the seventh-hour (or ninth-hour) pentatonic scale can be seen as a second anchor-group, in relation to the diatonic shell—triad X and the pentatonic scale are in the relationship of an anchor-within-an-anchor.

The diatonic shell consists of the eleventh hour steered by the ninth hour—or arranged more 'melodically', the eleventh hour steered by the sixth hour, steered by a perfect fourth (other steering interpretations are possible as well). The traditional power of the tonic is invested or implicit in the first, simplest version above of this anchor-shell. Here, C major is centrally placed in the lower shell (the 'major side'), together with its dominant and subdominant, and A minor, the relative minor and the other possible tonic, is similarly central in the inverse upper shell (the 'minor side').

However, a more central power lies in the tenth-hour anchor-group. If the tonic (either of them) represents 'home', then this anchor-group has traditionally represented quite the opposite—and historically its power proved greater than that of the tonic. Note

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[1] Or, more rarely, two intrinsic spins away, or a single note-shift away.

[2] Presumably it should be possible to find a mathematical proof that would settle this question one way or the other.

that this diatonic anchor-group, triad X, can be extended by adding a g# (octave) to form tetrad X, the chord of the diminished seventh, without any disturbance to the symmetry. No harmonic group was more instrumental than the diminished seventh in the disintegration of the old harmonic system, in breaking down the power of the tonic. For the diminished seventh has no less than *eight* possible resolutions in traditional harmony—four in its dominant minor-9th guise, and another four (much less harmonically 'directional') resolutions as a stepwise ornamental harmony, a quasi-'chromatic-appoggiatura' chord (except that it occurs more often on a weak beat)—see bar 2 of Chopin's A-flat *Ballade*, for instance(also the resolution it usually takes in blues and gospel music).[3] Due to the extreme ambiguity inherent in this unprecedented number of possible resolutions, or directions it may take, the diminished seventh, even within the boundaries of the old harmonic system, was responsible for some remarkable chromatic passages in which all sense of a tonic and of functional harmonic tonality have utterly disappeared (for example, the cadenza-like passage in Chopin's E-major *Etude* Op. 10, No 3—in which the eighth-hour tetrad, as the 'French sixth', also features as a significant tonic-countering agent).[4] Passages such as these are much more easily and sensibly analysed in tone-clock terms, rather than in any traditional functional-harmonic analytical terms.

My world-music anchor-form can hardly be proved 'right' (or 'wrong', for that matter)—and there may well be grounds for regarding it even as somewhat far-fetched. But whatever its merits ultimately, this interpretation does offer some new and, I think, interesting and relevant perspectives. In the light of the above, moreover, there may be some musical foundation, at least, for the idea (along Lévi-Strauss lines) that symmetry, mediation and complementarity do perhaps play a part in the structure and functioning of the collective unconscious mind.

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[3] In fact, the diminished seventh can resolve, in these ways, onto any of the 8 notes of Messiaen's Mode 2 (itself totally without implications as regards any traditional tonic)—so as well as being the principal destroyer of the old tonality, the diminished seventh might also be said to have pointed a way for the future. Moreover, as well as being the anchor-group of the above diatonic anchor-form, it is *also* the chromatic complement of Mode 2, and thus also the anchor-group of the 12-note anchor-form whose shell is Mode 2).

We might further consider Mode 2 and the diatonic scale as being next-door neighbours in the same **212** binding-network: the diatonic scale (aeolian mode) is **212** Binding-5 (**212** steered by a perfect 4th) and Mode 2 is **212** Binding-6 (**212** steered by a tritone)—and **212** Binding-7 (**212** steered by a perfect 5th) is the diatonic scale (*dorian* mode)! Mode 2 as binding-6 is centrally situated between the diatonic scale as binding-5 and binding-7.

Thus in terms of chromatic symmetry there are some very close connections indeed between Mode 2 and the diatonic system.

[4] See Schat(1993), p107. The concept of anchor-forms and steering-partners are my own ideas, which I passed on to Schat, and also to the Dutch composer André Douw(whose response suggested to me the idea of a simple hour-reversal as the most global form of steering-partner relationship). Schat and I later discovered the diatonic anchor-form at the same time, on opposite sides of the world. Our two letters crossed in the mail with the news, and he subsequently borrowed my example above, of the tenth hour in Chopin's E-major *Etude*.

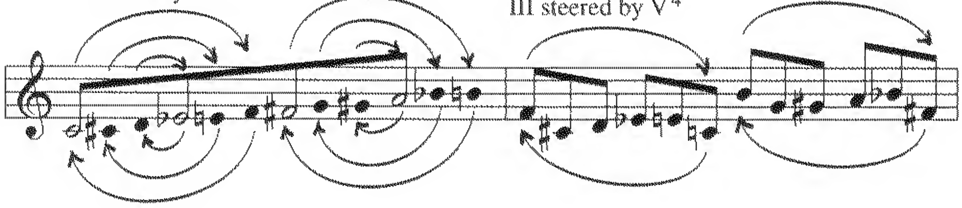
Appendix X: Intrinsic Spin in the Triadic Tonalities

Intrinsic spin can provide a useful systematic means of regrouping the 12-note triadic tonalities shown on p132. With this process, each hexad or half-hour assumes a centre-of-gravity, a sort of 'magnetic moment' or 'sense of itself' as a nucleus plus two shells, each of which can rotate within itself. When intrinsic spin is applied respectively to the two outer-shell notes, or to the two inner-shell notes, or to the two central nuclear notes, one finds oneself suddenly, by a very simple means, in a different chromatic tonality altogether, e.g.:

Hexad I^6

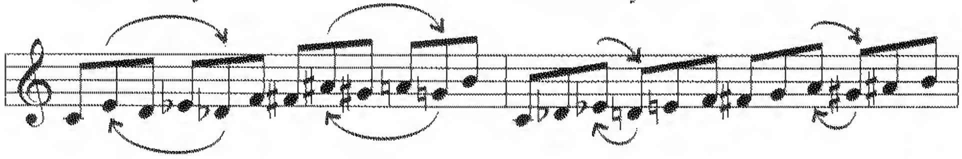
Original Tonality
I steered by X^4

Intrinsic Spin:
of Outer Shell
III steered by V^4



of Inner Shell
VI steered by V^4

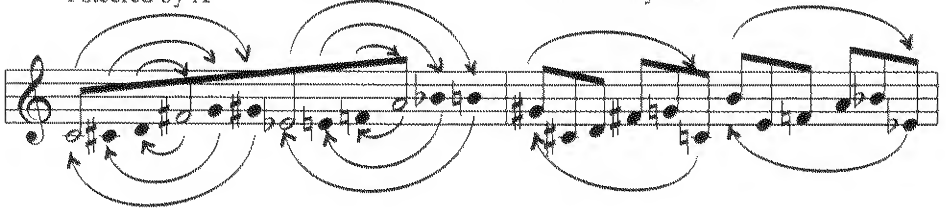
of Nucleus
II steered by $VIII^4$



Hexad **11411**

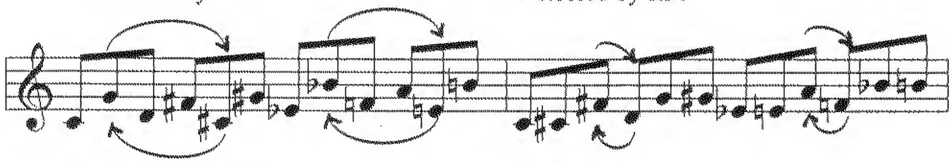
Original Tonality
I steered by X^4

Intrinsic Spin:
of Outer Shell
V steered by IIM^4



of Inner Shell
IX steered by X^4

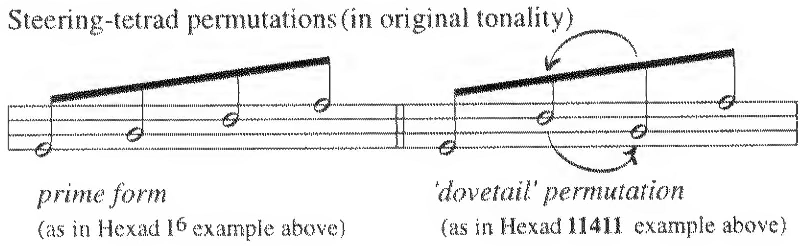
of Nucleus
V steered by IIM^4



In order to establish the patterns of note-exchange involved in these intrinsic spins, the following preliminary conditions must be present in the original tonality:

1. The triads must be in their prime form. (After the intrinsic spins are applied, the resulting new triads will no longer be in their prime form. However, this does not affect at all the implicit freedom of note- and triad-order that remains available when it comes to composing the actual music. No particular note-order or triad-order is implied: the intrinsic spins simply establish, at a deep-structural level, the *particular note-exchanges* that will create a new chromatic tonality.)

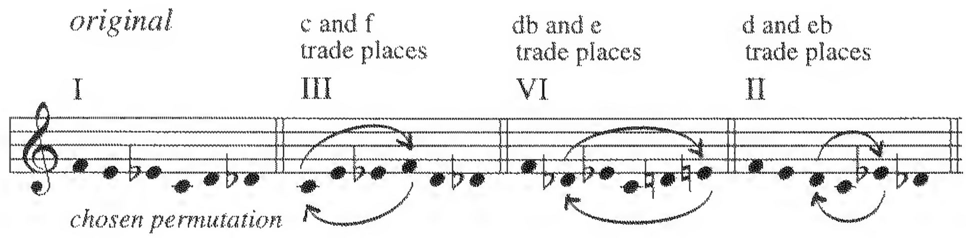
2. The four *steering-notes* must appear in one of the following two permutations:

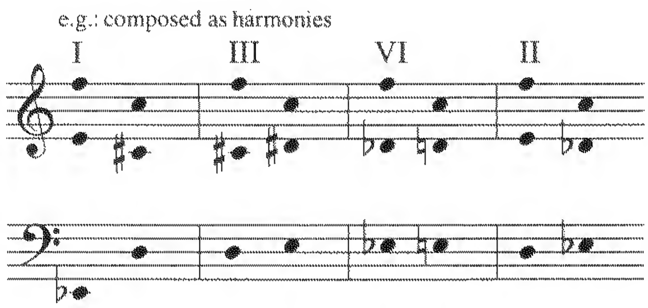


3. In the asymmetrical hours (those based on minor/major triads), each half-hour must contain one major and one minor triad (not necessarily in that order) in order for a *single* new tonality to be formed (except where the hexad concerned is 12312). In the main table, all the original hours (with the above exception) were first so arranged. If a dual tonality is desired, then one puts both major triads on one side and both minor triads on the other (except with hexad 12312, where this arrangement may still produce a single tonality).

In the first example on p135, a basic pattern of intrinsic spins is established for that form of the first hour in which each half consists of hexad I^6. With intrinsic spin of the outer shell, the notes c and f are interchanged (to form the third hour); with intrinsic spin of the inner shell, c# and e are interchanged (forming the sixth hour); and with intrinsic spin of the nucleus, d and eb are interchanged (forming the second hour). For the sake of simplicity, let us for the moment consider the first hexad only (though the pattern just observed, and all that follows, will apply equally to the second hexad or half-hour as well—and might also be understood as applying vertically instead of horizontally, for that matter).

How might this deep-structure appear on the manifest level? We can now take *any* permutation of two first-hour triads that will form hexad I^6. So let us put the second triad (in this case, eb, e, f) first, followed by the first triad. We have now quite removed ourselves from the original deep-structure note-order: the two triads are no longer in their prime form, and their two steering-notes are in reverse order. We can now take this newly-composed first-hour figure and apply to it the various note-interchanges already established by way of the earlier deep structure: i.e., c and f trade places, etc. This will give us three variants of the original first-hour figure in which, in each case, four of the six notes are unchanged (and in exactly the same position) as those of the original figure, but in all three cases a new chromatic tonality has resulted:





To my ear, such intrinsic spins possess a sort of subliminal inevitability, an inner consistency generated by the pervasive quality of a single chromatic hour (together, of course, with the fact that the same six notes are always involved in each half-hour)—even though there are obviously also various other passing inner triads present (such as III and II, for instance, in the chosen first-hour permutation). These particular permutations do not have quite the arbitrary, quasi-'accidental' feel that just 'any old permutation' to my ear often has (the sense of change simply for the sake of change).

The notes as composed may come in any order compatible with tone-clock freedoms, i.e., any subsequent transformations need only comply with the *pattern of interchange* established by the deep-structure intrinsic spins. They need not necessarily maintain the sort of note-order consistency that occurs in the variants shown above, but may come in *any* order that will maintain a triadic (and thus a chromatic-tonal) consistency.

The intrinsic-spin permutations can provide a useful way of moving beyond—but not 'too far beyond'—the note- and triad-permutations available within a single tonality. Their respective changes of colour are more marked than might be expected, given such a simple and restricted means, making this a useful and effective technique. By means of intrinsic spin, the composer can for instance quite easily give passing hints at related tonalities whilst essentially remaining within the original; or can create a 'tonality-continuum', for example, in which one tonality gradually transforms itself, via a chain of intrinsic spins, into another quite 'distant' one. Here is an organic and integrated process of 'modulation', which arises as a sort of tone-clock equivalent of modulation in the old classical system—not, one must stress, by way of any 'imitation' of the old modulation process, which (in theory, anyway) used a pivot chord common to both keys, but rather through a process which arises naturally from the properties of these chromatic symmetries. (Pivot groups can of course still be used, particularly to move from one steering to another of the same chromatic tonality; and a whole tonality may even be used as pivot, where it is common, as a related hour, to two different original tonalities. The pivot principle in these terms is no 'nostalgic return' to outworn tonal methods, but rather a fundamental and still perfectly viable technique with a well-established history. Even Boulez [one of the last one could accuse of any such 'nostalgia'] habitually uses tied pivot notes to connect adjacent harmonic cells—in Cycle I of *Le Marteau*, for example[1].)

The table on pp138-9 shows all of the 'hourly' transformations produced by regular (i.e., single-tonality) intrinsic spins. It is an expansion of the main tone-clock (TC) relationships shown in the IPF Chart, hexad section, 'Other Identities' column, pp49-55 (see also p24). For every tonality except the fifth hour, there is more than one set of regular intrinsic-spin possibilities available.

Every hexad named in the left-hand column of the table is the hexad formed by the two (prime-form) triads present in the first half (and generally also the second half) of the given tonality. I have indicated these hexads simply in order to make it easier for anyone interested to figure out what the deep-structure triads involved actually are. Thus, for example, under the second-hour heading, we read, for the first set of intrinsic spins, that the hexad concerned is I^6 (which for convenience we shall take as starting on c). Thus we now look for a permutation of I^6 which will give us both a minor and a major second-hour triad. There is only one possible permutation, namely, c, db, eb / d, e, f. (The other, complementary half of the tonality will be similarly formed from hexad I^6 transposed at the tritone.) Likewise under the same second-hour heading, the hexad **21112** is shown as the basis for the second set of intrinsic spins, so once again we look for a permutation of this hexad which will give us both a minor and a major second-hour triad. Again, there is only one possible version, namely the prime form of the hexad, c, d, eb / e, f, g, with the remaining half of the tonality once again being similarly formed from hexad **21112** transposed at the tritone.

Where (under other hour-headings in the table) the hexad concerned is **11411**, **13131** or VI^6, the second, complementary half of the tonality will still consist of the same hexad transposed, but not at the tritone (for obvious reasons) in the case of **11411** and

[1] See Koblyakov (1990).

ORIGINAL TONALITY	INTRINSIC SPIN OF		
	OUTER SHELL	INNER SHELL	NUCLEUS
FIRST HOUR			
I steered by X^4 Hexad I^6	III steered by V^4	VI steered by V^4	II steered by $VIII^4$
I steered by X^4 Hexad **11411**	V steered by IIM^4	IX steered by X^4	V steered by IIM^4
SECOND HOUR			
II steered by $VIIIm^4$ Hexad I^6	VI steered by V^4	III steered by V^4	I steered by X^4
II steered by $VIIIM^4$ Hexad **21112**	IV steered by $VIII^4$	VII steered by $VIII^4$	VI steered by X^4
II steered by $VIII^4$ Hexad 12312	----------------	XI steered by $VIII^4$	----------------
THIRD HOUR			
III steered by Vm^4 Hexad I^6	I steered by X^4	VI steered by V^4	II steered by $VIII^4$
III steered by VM^4 Hexad **13131**	XI steered by VI^4 or $VIII^4$	XII	IV steered by VI^4 or $VIII^4$
III steered by $VIIm^4$ (or IXm^4 or IX^4) Hexad **13131**	XI steered by VI^4	XII	IV steered by VI^4
III steered by V^4 Hexad 12312	----------------	XI steered by $VIII^4$	----------------
FOURTH HOUR			
IV steered by VI^4 Hexad **13131**	XII	XI steered by VI^4	III steered by $VIIm^4$ (= IXm^4)
IV steered by $VIIIM^4$ Hexad **13131**	XII	XI steered by $VIII^4$	III steered by V^4
IV steered by $VIIIm^4$ Hexad **21112**	II steered by $VIII^4$	VII steered by $VIII^4$	VI steered by X^4
IV steered by $VIII^4$ Hexad **11411**	----------------	IX steered by X^4	----------------
FIFTH HOUR			
V steered by IIM^4 Hexad **11411**	IX steered by X^4	V steered by IIM^4	I steered by X^4
SIXTH HOUR			
VI steered by V^4 Hexad I^6	II steered by $VIII^4$	III steered by V^4	I steered by X^4
VI steered by X^4 Hexad **21112**	VII steered by $VIII^4$	IV steered by $VIII^4$	II steered by $VIII^4$
VI steered by V^4 Hexad VI^6	VII steered by $VIII^4$	IV steered by $VIII^4$	II steered by $VIII^4$
VI steered by V^4 Hexad VI^6	VIII steered by $IIIm^4$	XII	VIII steered by $IIIm^4$
VI steered by X^4 Hexad VI^6	VIII steered by $IIIM^4$	XII	VIII steered by $IIIM^4$

ORIGINAL TONALITY	INTRINSIC SPIN OF		
	OUTER SHELL	INNER SHELL	NUCLEUS
SEVENTH HOUR			
VII steered by VIIIm^4 Hexad **21112**	VI steered by X^4	IV steered by VIII^4	II steered by VIII^4
VII steered by VIII^4 Hexad 12312	----------------	XI steered by VIII^4	----------------
VII steered by VIIIM^4 Hexad **22122**	XI steered by VIII^4	IX steered by X^4	VI steered by V^4
EIGHTH HOUR			
VIII steered by IIIm^4 Hexad VI^6	XII	VIII steered by IIIm^4	VI steered by V^4
VIII steered by IIIM^4 Hexad VI^6	XII	VIII steered by IIIM^4	VI steered by X^4
VIII steered by IVM^4 (=XIM^4) Hexad VI^6	XII	VIII steered by IVM^4 (=XIM^4)	VI steered by V^4
NINTH HOUR			
IX steered by IIm^4 Hexad **11411**	V steered by IIM^4	V steered by IIM^4	I steered by X^4
IX steered by VIIIm^4 Hexad **22122**	VI steered by V^4	XI steered by VIIIm^4	VII steered by VIIIM^4
IX steered by X^4 Hexad **22122**	XI steered by VIII^4	VI steered by V^4	VII steered by VIII^4
ELEVENTH HOUR			
XI steered by VI^4 or VIII^4 Hexad **13131**	XII	IV steered by VI^4 or VIII^4	III steered by V^4 or VIIm^4 (=IXm^4)
XI steered by VI^4 Hexad **22122**	VII steered by VIII^4	IX steered by X^4	VI steered by V^4
XI steered by VIII^4 Hexad 12312	----------------	VII steered by VIII^4	----------------
TWELFTH HOUR			
XII steered by I^4 Hexad **13131**	IV steered by VI^4	XI steered by VI^4	III steered by VIIm^4 (=IXm^4 or IX^4)
XII steered by V^4 Hexad **13131**	IV steered by VIII^4	XI steered by VIII^4	III steered by V^4
XII steered by X^4 Hexad **13131**	XI steered by VIII^4	IV steered by VIII^4	III steered by V^4
XII steered by IIM^4 Hexad VI^6	VIII steered by IIIM^4	VIII steered by IIIM^4	VI steered by X^4
XII steered by VIIm^4 (=IXm^4 or IX^4) Hexad VI^6	VIII steered by IVM^4 (=XIM^4)	VIII steered by IVM^4 (=XIM^4)	VI steered by V^4

VI^6, and not necessarily at the tritone, in the case of **13131**. In these cases, the symmetrical tetrad (shown in the left-hand column) that *steers* the whole tonality will provide the essential clue as to the disposition of the triads in the second half.

Where the hexad concerned is 12312, the second, complementary half will consist not of the same hexad transposed but of its inversion 21321 (in one of two possible transpositions).

Note: hexads **13131** and VI^6 are exceptional in that *all* of their possible permutations (and not just the present intrinsic-spin permutations) are *always* in a single chromatic tonality.

The table demonstrates clearly the fundamental connection between Babbitt's six 'all-combinatorial' hexads and the tone-clock tonalities. These six hexads are respectively I^6, **21112**, **22122**, **11411**, **13131** and VI^6. Together with the hexad 12312, these six hexads form the 'half-hour' basis for every possible version of a chromatic tonality to which intrinsic spin can be applied so as to form a whole new chromatic tonality (as opposed to a dual tonality). All are Class 1 hexads (symmetrical hexads whose chromatic complement is the same transposed) apart from 12312, whose chromatic complement is its inversion (and which is also exceptional as the one and only *asymmetrical* chromatic group of limited transposition).

Tonalities thus related by intrinsic spin can also work together as dual tonalities, with the first half-hour in one tonality, and the second half-hour in a different, related tonality.

Where the same tonalities with the same steerings apparently keep on cropping up in the table, their relative *transpositions* are usually different (except for XII, which has no real transpositions), suggesting further compositional possibilities again.

Note 1: Wherever XII is formed in the table, no steering is shown, since any of its many steerings may be taken (with the twelfth-hour triad, any of its notes may without further ado act as the steering-note).

Note 2: More complex forms of intrinsic spin (for instance, of two shells simultaneously: outer + inner, inner + nuclear, or outer + nuclear) can be ignored, since they yield the same results as shown, i.e., a simultaneous rotation of outer + inner shells has the same effect as a rotation of the nucleus, only the resulting triads are in the opposite order, and so on (which of course is negligible when the triads of a twelve-note tonality can in principle come in any order).

Transformation by Tritone Transposition of Individual Notes

Babbitt (1955) observed that four of the six 'all-combinatorial' hexads are nicely inter-related, in that through successive tritone transposition of their individual notes they become transformed into one another. They are also transformed, however, into hexad 12113 or its inversion—another exceptional group, we recall, as the only asymmetrical hexad whose chromatic complement is the *same* transposed (at the tritone). The chart below shows these deep-structural relationships, which can easily be used to create new inter-relationships (now through tritone transposition of successive notes of the hexad) between many of the chromatic tonalities shown in the main table. Hexads **11411**, VI^6 and 12312 do not feature here, since each of these already consists of a triad plus its tritone transposition.

Hexad	Transposition at the tritone of:					
	1st note	2nd note	3rd note	4th note	5th note	6th note
I^6[c]	I^6[c#]	**21112**[c]	12113[c]	31121[a]	**21112**[b^b]	I^6[b]
21112 [c]	I^6[d]	31121[c]	**22122**[c]	**22122**[b^b]	12113[b]	I^6[c]
22122[c]	**21112**[d]	12113[e]	**22122**[f]	**22122**[g]	31121[a]	**21112**[c]
13131[c]	31121[c#]	12113[e]	31121[f]	12113[g#]	31121[a]	12113[c]
12113[c]	**21112**[c#]	31121[c]	**13131**[c]	**22122**[g#]	31121[g#]	I^6[c]

The one hexad common to each set of transformations above is also the one hexad which, when itself transformed, reproduces both itself *and* all four of the other hexads involved, namely, 12113 or its inversion 31121. This hexad, however, does not appear anywhere in the main table, even though it *is* a tone-clock hexad. The reason is that intrinsic spins of the

only triadic tonality based on hexad 12113—namely, the fourth hour (either four minor or four major triads) steered by X^4—produce no single new tonality (though they do produce a dual tonality).

The fourth hour steered by X^4 is the only 12-note triadic tonality that does not appear anywhere, in any form, in the main table. Babbitt's tritone-transposition process thus provides a potential gateway between this one exceptional 'missing' steering and the other triadic tonalities. Triad IV, we recall, is also the only asymmetrical triad that can be steered by X^4 so as to form a regular 12-note tone-clock tonality: all the other triads so steered by X^4 are symmetrical, or incipiently symmetrical in the case of triad IX—i.e., triads I, VI, IX and XII. Moreover, every other asymmetrical triad (i.e., II, III, V, VII, VIII and XI), when similarly steered (i.e., either as four minor triads or as four major triads) by X^4, always forms Messian's Mode 2 (the second-hour subscale)—a property providing a further 'gateway', between the subscales or modes of limited transposition and the tone-clock tonalities.

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De Groot, Rokus. 'The Wheels of the Tone Clock: The Musician as Clockmaker.' *KeyNotes* 19 (1984): 7-17. De Groot has here already observed the role played by intrinsic spin (which he calls 'intertwining') of the hexad nucleus, in transformations of the 12-note triadic tonalities. For him, 'intertwining' is a two-part operation, however, a cross-exchange of two notes, whereas I prefer to see it as a single operation or 'movement', a spin (as I say) like the 'half-turn of an oven switch'.
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